Finding the Hausdorff Dimension of the Kigami Triangle

Paul Tippet, Department of Mathematics,
Australian National University

This was a project in fractal geometry, under the supervision of Prof. Michael Barnsley. Some assistance from Prof. John Hutchinson was sought as well.

The aim of the project was to find the fractal dimension of the fractal set known as the Kigami triangle.

First a review of the literature on the subject was in order. In particular papers by Falconer, Teplyaev, Teufl, Barnsley and Freiberg were collected. Following that the first couple of chapters from Strichartz's book *Differential Equations on Fractals: A tutorial* were studied.

Elma Teufl, as part of his recent PhD thesis, found an upper bound for the Hausdorff dimension of the Kigami triangle – about 1.32. However he found that by using a quick computational estimate via box counting he got a much lower estimate. So the aim in this project was to improve on either (or both) of the Hausdorff dimension or box counting (Minkowski-Boulegand) dimension estimates and in particular to be confident about the accuracy of the estimates.

Our attempt at improving the estimate for the Hausdorff dimension wasn't successful. This involved trying to find the limiting growth rate of the length of the boundary (of the fractal) via piecewise linear approximations, as successive iterations of the transformations were carried out. However a useful theorem gives us that the box counting dimension is greater than or equal to the Hausdorff dimension, so by successfully computing an estimate for the box counting dimension, we actually found an (accurate) estimate for the upper bound of the Hausdorff dimension.

To find a very good estimate for the box counting dimension we employed a combination of two algorithms to draw the attractor – the deterministic algorithm and the chaos game. After some careful tweaking of parameters we could bring the error in the image to less than 10 parts in 100,000,000. And this error was only likely to have influence on the very finest of resolutions which we eventually excluded to get an accurate data sample. From the image the usual process for finding the box counting dimension was followed, including fitting a linear model and minimizing the least squares error.

As a result, we had computed an estimate for the box counting dimension and hence a new upper bound for the Hausdorff dimension of the Kigami triangle – 1.1873.

This project was beneficial mainly as an exploration into fractal geometry. Much attention was given to the procedural programming – creating efficient and accurate code – so some experience was gained here as well.

Paul was an AMSI/MASCOS vacation scholar for 2007-8.