



INTERNATIONAL CENTRE  
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EDUCATION IN  
MATHEMATICS

### **Mathematical Physics**

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The project to which I was assigned during the 2004-5 vacation period involved reading and familiarising myself with the mathematics of quantum statistical mechanics as well as producing and explaining the derivation of two key equations from one-dimensional solvable systems: namely the Lieb-Liniger and the Yang-Yang equations. The mathematics required to completely understand the processes dealt with in these classic papers was largely comprised of integral equations and extended use of Lagrange multipliers.

The AMSI Summer School also proved to be an excellent supplement to the summer scholarship [and I heartily recommend this to future students], as the courses in measure theory and combinatorial matrices provided essential background machinery for my project. Indeed, direct applications of the dominated-convergence theorem were needed in order to accurately justify some of the steps in the derivation of the Yang-Yang equation.

Specifically, the equations deal with the ground-state energy and wave function of a large system of bosons (Lieb-Liniger) and the entropy and free energy per particle (Yang-Yang). These equations are fundamental to the further understanding of mathematical physics in the sense of quantum effects observed on a macroscopic scale. Recent experimental results for quantum gases of bosons and fermions have motivated a renewed interest in this field, which is certain to remain an interesting branch of mathematical research for years to come.