Visualising Spacetime Geometry: Black Holes and Matter Flow

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The Einstein equations govern how space-time and matter interact. These are a set of coupled non-linear partial differential equations and analytic solutions are difficult to find. However, under conditions of high symmetry there exist well-known solutions - the most famous being the Schwarzschild solution. The Schwarzschild solution is a vacuum solution and spacetime is assumed to be spherically symmetric about a point. In the Schwarzschild solution, spacetime is foliated by 2-spheres and there exists four Killing vector fields due to the spherical symmetry.

My project involves studying the spherically symmetric solution of the Einstein equations and the techniques used. I then analyse the geometry of these solutions to convert the information into a visual form, in hopes of a more intuitive understanding of its often strange features, such as the event horizon of black holes.

The Schwarzschild solution is usually written in terms of the Schwarzschild metric:

$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2$$

where t is the timelike coordinate orthogonal to the spacelike "radius" coordinate labelling the radius of the 2-spheres and the last term is the metric on the 2-sphere.

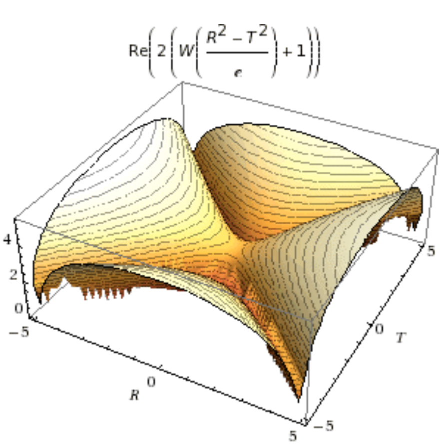
At r=2GM, the metric becomes appears to become singular. This hypersurface is not a real singularity but rather an "event horizon" where light cones tip over and even light cannot leave. The singularity-like behavior of the metric in these coordinates can be interpreted as an infalling object that appears to fall forever for a faraway observer.

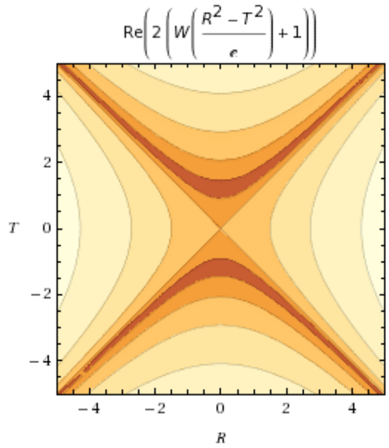
However, by changing to Kruskal coordinates, the metric takes the form:

$$ds^2 = \frac{32G^3M^3}{r}e^{-r/2GM}(-dT^2 + dR^2) + r^2 d\Omega^2$$

where T and R are the new timelike and spacelike coordinates respectively. In this coordinate system, the only singularity appears to be at the origin, ie. the black hole.

One of the undertakings during my project was to visualise this set of coordinates with the following result:





The contour map on the right is conventionally known as a Kruskal diagram and the left diagram is a three dimensional plot of R and T as the independent variables and the radius r as the dependent variable. The region at the top represents the region between a black hole and its event horizon. The region to the right is regular spacetime (asymptotically flat). The other two regions are unrealistic features such as a while hole and a separate universe.

I learned a lot about general relativity and black holes during my summer vacation scholarship. The icing on top of the cake was my chance to participate in the AMSI-CSIRO Big Day In in Sydney where I gave a presentation on my project. It allowed me to sum up what I had done and communicate it to an audience. I also got to meet fellow scholars and hear about all the interesting things that they studied over summer.

Thanks go to my supervisor Robert Bartnik for his patience and to AMSI for their generous funding.

Joseph Chan received a 2010/11 AMSI Vacation Research Scholarship

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