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**Experiments on the Numerical Simulation of the Outer Solar System**  
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The outer solar system is a typical  $N$ -body problem. When the mass, the initial velocities and positions of each body are given, this system can be integrated by solving a system of first-order ordinary differential equations using numerical integrators. This is called a numerical simulation of the outer solar system.

We have implemented several conventional integrators and geometric integrators, then numerically simulated the outer solar system over a long time span, and compared the simulation results from using different integrators in terms of long-term stability and accuracy.

We have verified the conclusion that geometric integrators are superior to conventional integrators for long-term simulation, because they can preserve some qualitative behaviour of the system such as symplectic structure, or energy.

To show the correctness of the implementation, we have written a test program for solving the Kepler problem by re-using the integrators implemented for the outer solar system, since we know the exact analytical solution for the Kepler problem.

We have also investigated and applied the technique of compensated summation to our simulation program to reduce the round-off error of the numerical computations, and found that this technique could save us two significant digits for the simulation results without any notable influence on increasing the computing complexity of the whole program.

From the experiments, we believe that a higher-order symmetric composition method such as the eighth-order McLachlan is very suitable for long-term simulation of the outer solar system due to its excellent long-term accuracy and stability.