# Important Inequalities and Interpolation

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The aim of my project was to study some of the most important inequalities in mathematical analysis. This included a study of interpolation method theorems regarding the boundedness of linear operators. I also considered applications. These inequalities and theorems were studied with respect to Lebesgue spaces $L^p(E)$ which are function spaces in mathematical analysis. For this project I investigated several texts. The main ones were: Inequalities by Hardy, Littlewood and Pólya; Trigonometric series by Zygmund; Measure and Integral by Wheeden and Zygmund.

The first inequality I studied was Hölder's inequality which bounds the product of two functions. I then studied Young's inequality which bounds the convolution of two functions. Lastly I studied the Hausdorff Young inequality. This inequality provides a bound for the Fourier transform of the function f and can also be used to provide a bound for the Fourier series of f.

Next I considered two interpolation theorems. The first was the Reisz Thorin interpolation theorem. This theorem takes the boundedness of an operator in $L^p$ and in $L^q$ and extends this to a statement of boundedness for that operator in all $L^r$ for $p < r < q$. Next I looked at the Marcinkiewicz interpolation theorem. This theorem considers two types of boundedness, weak type and strong type. If an operator is of weak type for $L^p$ and $L^q$ then the operator is of strong type for $L^r$ , $p < r < q$.

As an application I studied the absolute convergence of the sum of the Fourier coefficients for a given function. If the sum of the Fourier coefficients was absolutely convergent we could approximate the original function by a partial sum. The first application result showed that if a derivative of a function was in $L^p, (p > 1)$, it was possible to show the Fourier coefficients sum was absolutely convergent. Two more results showed that the Fourier coefficients sum is absolutely convergent if the function is Lipshitz of degree $\alpha > \frac{1}{2}$ or if the function is Lipshitz of degree $\alpha > 0$ and is bounded by some constant.

At the ‘Big Day In’ event for 2010 I presented proofs for a couple of the inequalities and theorems listed above before explaining the applications. This event was very useful, as it allowed me to practice my own presentation skills as well as giving an opportunity to see the research and interests of other vacation scholars in many areas of mathematics. Overall the experience gave me the opportunity to learn skills involved in researching and applying mathematics in an area of interest.

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