Numerical Solutions to Elliptic and Parabolic PDEs using Finite Elements and Wavelets

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In my project I looked at two modern techniques for solving these sorts of problems. The finite element method is a classical technique where instead of searching a full function space for a solution you look for an approximate solution in a finite-dimensional subspace. With the wavelet-based method, developed by Cohen, Dahmen and DeVore in 2001, you can set up an infinite linear system for the *exact* solution, and solve this numerically. The advantage is that you don't have inherent errors from choosing a subspace – this method is adaptive in the sense that it chooses the best subspace for you to work in (and successive iterations improve this choice – in the limit it approaches the exact solution).

I proceeded by reviewing some of the literature on these two techniques where they are derived and their convergence properties proven. The majority of my work was focused in developing implementations of these techniques for simple problems in order to verify that these properties hold in the real world – it turns out that the results indeed closely follow the theoretical results. The basis of these implementations I hope could provide useful starting points for more complicated, perhaps multidimensional problems.

This was a very beneficial experience as I proceed through my undergraduate degree with a view to Honours and probably postgraduate research. The generous support of AMSI and CSIRO in allowing me to experience life as a researcher has been invaluable to my development as a mathematician. Of course I also have to thank my supervisor Markus Hegland and all of the other members of the advcomp group at ANU for putting up with me for the whole summer, it was a pleasure.

Lindon received an AMSI Vacation Scholarship in December 2009 See: www.amsi.org.au/vs10.php