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Non-linear Network Dynamics: The Groupoid Formalism

Daniel Sutherland, School of Mathematical and Physical Sciences, The University of Newcastle

Along with two other students from the University of Newcastle, Christopher Wood and Michael Rose, I studied a paper published in the Bulletin of the American Mathematics Society by Martin Golubitsky and Ian Stewart entitled "Nonlinear Dynamics of Networks: The Groupoid Formalism". The paper concerned itself with the existence of certain periodic states that the network could occupy.

Networks were formally described as being comprised of cells (or nodes), edges (or one-way arrows) between the cells, an equivalence relation between cells and between edges, and two mappings that return the "head" or "tail" of an arrow. The cells of the network represented state variables of the system, and the edges represented some form of dynamic interaction between certain cells. In this way, the state of a cell at any point in time can be described by a vector in $\mathbf{R}^n$. The time evolution of these states allows certain cells to be characterised in certain ways. The paper describes how cells are synchronous when they behave periodically with the same period and no phase shift, or how a network is phase-locked when all the cells experience dynamics with the same period, but each is a fixed phase apart.

In the early sections of the paper, it is shown that global symmetry of a network naturally gives rise to these dynamic phenomena in such a network. It is further shown that whilst symmetry guarantees the existence of these things, it is not necessary for their occurrence, as some clearly asymmetric networks give rise to similar behaviour. This is the motivation behind most of the paper; constructing a notion of symmetry that's not as restrictive as "global symmetry", and yet is both necessary and sufficient for the existence of certain phase relations.

The paper continues to introduce further notation from both a graph-theoretic nature and a dynamical system nature to reach its conclusions. A new form of "symmetry" is found, and it is shown how certain "complicated" networks can be compressed down to form a "quotient" of the original network. Results relating a network to its quotient abound, including that phase relations transcend this operation.

I thoroughly enjoyed working through the unfamiliar territory that this paper provided, and coming to understand the things I do now about dynamical systems. This experience has further encouraged me to continue my mathematical pursuit, and will hopefully equip me to tackle future mathematical studies. I am grateful to Newcastle University and AMSI for giving me the opportunity to partake in this scholarship, and would again participate if given the chance.

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See http://www.ice-em.org.au/students.html#scholarships2007