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Wave equations and symmetry in relativity

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During my summer vacation scholarship I have worked with Dr Jonathan Kress, studying Killing tensors and the symmetries to which they give rise in Maxwell's and Laplace's equations.

I began by reading part of "Advanced general relativity" by J. Stewart where I learnt about Lie derivatives of tensors, Killing's equation and Killing vectors. I went on to prove the result that the Lie derivative of a metric **g** with respect to a Killing vector field is zero. That is, Killing vector fields give symmetries of the manifold. I also went on to solve Killing's equation with a Lorenzian metric. After reading a section of Flanders' "Differential forms" regarding the Hodge star, I was able to see the way Maxwell's equation can be written using differential forms. I proved that the exterior derivative commutes with the Lie derivative and that the Hodge star commutes with the Lie derivative with respect to a Killing vector field, from which it follows that the Lie derivative with respect to a Killing vector field gives a symmetry of Maxwell's equation.

I then learnt how Killing vectors can be generalised to Killing tensors and then further generalised to conformal Killing tensors. Using Maple, I was able to solve the conformal Killing's equation in flat space and find all the conformal Killing vectors that correspond to a Euclidian metric. I then studied Michael Eastwood's paper "Higher symmetries of the Laplacian" and verified, with the help of Dr Kress, the result he gave regarding how, in flat space, conformal Killing tensors of rank one and two give rise to symmetries to Laplace's equation. I also looked at N. Kamran and R. G. McLenaghan's paper "Separation of Variables and Symmetry Operators for the Conformally Invariant Klein-Gordon Equation on Curved Spacetime". The last result in the paper is also about conformal Killing tensors giving symmetries to Laplace's equation, but in curved space. I simplified the result to flat space and tried to find a connection between Kamran and McLenaghan's result and Eastwood's result. It was clear that Eastwood's result was more general, however, I did not manage to find the exact connection between Kamran and McLenaghan's result and Eastwood's result even in cases that should coincide.

Finally, in preparation for the "Big Day In", an event where I have to give a presentation regarding the work I did over the summer, I gained knowledge on how to use Latex.

Boris received an ICE-EM Vacation Scholarship in January 2005.
See www.ice-em.org.au/students.html#scholarship.