INTERNATIONAL CENTRE
OF EXCELLENCE FOR
EDUCATION IN
MATHEMATICS

Modern advances in the invariant subspace problem

Charles Qin, School of Mathematics, University of New South Wales

The aim of this research vacation was to study some of the progresses made on the invariant subspace problem over the past century. The entire project was divided into two parts, where one part required me learning the basic ideas and concepts involved in modern analysis while the other part had strong emphasis on the understanding of some of the major results and proofs associated with the invariant subspace problem in both the Hilbert Space and the Banach Spaces.

In the first two and half weeks, I read selected sections from the text books *Introductory Real Analysis*, by A. N. Kolmogorov and S. V. Fomin, and *A Course In Modern Analysis And Its Applications*, by G. Cohen to establish some foundation for the project. The first idea covered was the definition of metric spaces, as well as other important properties associated with it including dense, separable, closed, totally bounded, complete and compact spaces. The various types of convergences including convergence in norm, strong topological convergence and weak topological convergence were discussed. Some essential linear operator theory was also inspected in the first part of the project. The main concepts looked at included continuity, compactness and the different types of convergence of linear operators.

The second three and half weeks of the research vacation was spent on exploring various breakthroughs made on the invariant subspace problem. Some of the papers I read included *The Invariant Subspace Problem*, by H. Radjavi and P. Rosenthal, *Remark On Lomonosov's Lemma*, by W. Longstaff, and *The Present State And Heritages Of The Invariant Subspace Problem*, by B. Yadav. Major results considered included Von Neumann's Theorem, stating that all compact operators have non-trivial invariant subspaces, as well as Enflo's ideas of constructing a Banach Space with a linear operator without any non-trivial invariant subspaces. Lomonosov's Theorem, its proof and its various implications were studied in great details. It was noted that as a result of Lomonosov's Theorem and contributions from another mathematician, named Daughtry, that an equivalent problem to the invariant subspace problem was to show that any linear operator A has a non-trivial invariant subspace if rank(AK − KA) is two for some non-zero compact operator K.

Overall, the six weeks project has been extremely rewarding. It has given me great insights to mathematical research as well as presenting me with some essential knowledge to introductory analysis and operator theory.

Charles received an ICE-EM Vacation Scholarship in January 2005.
See www.ice-em.org.au/students.html#scholarship.