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Global cross-entropy optimization principles
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During my vacation scholarship I had the opportunity to explore the ground-breaking discoveries made by Claude de Shannon. I was surprised that I knew so little about a scientist who made the information era possible. Shannon's ideas brought unity to a variety of mathematical fields. His concept of Entropy connects Statistics with Information Theory and Quantum Physics. Shannon's groundbreaking idea was that information is not a mysterious abstract notion but a concrete entity that can be measured and modelled just like we measure the temperature on a cold day and try to predict its value on the next day.

Shannon and his fellow researchers realized one can use distance measures designed specifically to measure discrepancy between two probability densities to measure information (or structure/pattern) content, disorder, uncertainty, diversity, complexity, similarity, dependence and bias, within a given probabilistic or information system. They called such measures *Cross Entropy* measures.

There are many applications of these measures in all fields of science. During my scholarship I was interested in the application of these Cross Entropy measures to estimation and optimization. My exploration focused on the major link between Information theory and statistics. The link is given by this simple observation:

1. A *posteriori* distribution p
2. A *a priori* probability distribution q
3. A generalized convenient measure of *Cross Entropy* (i.e. distance metric between two probability densities) M
4. A set of constraints C connecting the observable reality with the probabilistic entities.

Then under appropriate conditions the fourth entity can be determined. This observation was the centrepiece of my scholarship investigations. The most exciting part of the scholarship was applying the link to analyze probabilistic systems such as rare event simulation and stochastic optimization.