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Constructing a Special Class of Partial Latin Square

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Latin Squares are n by n arrays in which each row and column contains the numbers 1 to n exactly once. Similarly, Partial Latin Squares are n by n arrays in which each row and column contains the numbers 1 to n at most once. The process of "filling in" empty values in a Partial Latin Square to transform it into a Latin Square is known as completion. There is a great deal of work being done on Partial Latin Squares problems, most of which is in the specific properties individual squares possess.

For my AMSI Summer Scholarship at the University of Queensland under the supervision of Professor Peter Adams, I instead chose to look at computational methods by which every Partial Latin Square of a given size that fulfils a given criteria may be constructed. In particular, my goal was to re-do and verify work previously commenced by Peter Adams and Trevor Pickett in examining Partial Latin Squares that I have dubbed "Prematurely Critical" Partial Latin Squares. These squares have the property that they themselves cannot be completed, but removing any one element produces a partial square with exactly one completion and removing any two elements produces a partial square with multiple completions.

A large part of my work involved ways of winnowing down the number of Partial Latin Squares that need to be considered. Since the number of (Partial) Latin Squares of a given size grows super-exponentially, this is extremely important if any program is to complete execution in a reasonable amount of time. Since many Partial Latin Squares may be grouped into what is known as isotopy classes - where all the members share the same underlying structure - a large part of this winnowing was aimed at ways of categorising which isotopy class a Partial Latin Square belonged to without requiring comparison with the others. This allows the program to construct only a representative member of each isotopic class, thus greatly reducing the run-time costs.

To implement such a program, I learned the functional programming language Haskell, which has a very mathematics-like syntax, and compares favourably with C in terms of efficiency. I also had to spend a lot of time learning how to optimise programs, both by profiling and improving the code and by compiler optimisations. As an example, I managed to drop the run time to produce order four Partial Latin Squares from two minutes down to approximately fifteen seconds by these optimisations.

Ultimately however, my algorithms still require a lot of work. It took over five days to run for order five Partial Latin Squares. As such, it would take too long to verify the original conjecture that no Prematurely Critical Partial Latin Squares of order six or higher exist. For orders two and three, one partial square was found, and five and six partial squares for orders four and five respectively. Out of these latter two orders, three of each were not known previously.

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See http://www.ice-em.org.au/students.html#scholarships2007