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Investigation of Class 1 type Models for Calcium Oscillations
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This study sought to investigate the characteristics of two existing models for intracellular calcium dynamics. Both the Atri model and Li-Rinzel models were implemented into the software program XPPAUTO enabling comparison by numerical techniques.

For given parameter values, two distinct types of calcium oscillations were produced by the model. Relaxation oscillations consist of repeated calcium spiking, whereas mixed-mode oscillations (MMOs) consist of several small 'wiggles' followed by one or more large spikes. The parameter values required for oscillations of either type were of particular interest.

IP₃, an important chemical mediator in the cell, is closely related to the dynamics of calcium. Both of the two models examined incorporated the dynamics of IP₃ concentration, p . The behaviour of p was described in both models by the equation:

$$p' = v - \beta p$$

Thus p , after some transient time, to approach the steady state v / β . The values of v and β which gave rise to relaxation oscillations or MMOs were investigated. These regions were found to depend upon the ratio v / β .

Non-dimensionalisation of the two models was performed to enable a better understanding of their operation. This led to the realisation that the Li-Rinzel model could be qualitatively made to resemble the Atri model by variation of a single parameter. Furthermore, both models could be made to resemble Class II type models by variation of other parameters to change the time scales of the respective equations.

In addition, the behaviour of both models in response to IP₃ pulsing was investigated. This corresponds to the artificial experimental injection of IP₃ into cells, having an effect upon the resulting calcium oscillations. Pulsing by IP₃ has previously been used in studies as a means of determining the dominant mechanism being of a Class I or Class II type. The effects of the pulsing strength and position in the oscillation cycle were investigated by measuring the subsequent delay in oscillations as a ratio of the normal oscillatory frequency. It was found that for values of v slightly above that required for MMOs, the delay became sensitive to the strength and position of the pulse. Care must therefore be taken when interpreting results from pulsing in this region.

This vacation project enabled me to have a taste of what mathematical research consisted of. It was especially enjoyable as it brought together with maths the different subject areas of chemistry, molecular biology and physiology, each for which I have a great interest. The project also made me appreciate the many hours of work that underlies often deceptively simple published results.