How many invertible n x n Matrices with k nonzero entries over a Finite Field?

Amelia Dale
School of Mathematics and Statistics
The University of Sydney

Define a function **m**(k, n, q) as the number of n x n invertible matrices with k nonzero entries over a finite field of q elements. This project investigated the formulae and behaviour of **m**(k, n, q). In the first two weeks there was an exploration of the research done on matrices over finite fields. A great deal was learnt, but little could be found relating to this specific question.

I became more familiar with combinatorics. By looking at the different possible arrangements of nonzero and zero entries, formulae was found for when k=n, n+1, n+2. In the special case when q=2 there exist no invertible matrices such that k>n^2-n+1 and formulae was found for **m**(k, n^2-n+1, 2) and **m**(k, n^2-n, 2).

Both computer (the Sydney University algebraic system Magma) and paper methods were used to look at the shape of **m**(k, n, 2), it is not quite a symmetric function, but does peak near the middle, close to k=n^2/2.

I was a bit disappointed that I was not able to find a general formula, whether it be messy, inductive or otherwise for **m**(k, n, q) or even **m**(k, n, 2). I sometimes found it frustrating counting the many possible cases of linearly independent arrangements of vectors. However, the opportunity to experience mathematics in a research environment, rather than a lecture theatre was not to be missed. I would like to thank AMSI and Sydney University for giving me this wonderful opportunity, as well as Anthony Henderson for his support.

References

W. Bosma, J. Cannon and C. Playoust, 'The Magma algebra system. I. The user language', *J. Symbolic Computing* 24(3-4) (1997) 235--265.

J. Fulman, 'Random matrix theory over finite fields', *Bulletin of American Mathematical Society* 39(1) (2001) 51--85.

J. Kahn and J. Komlos, 'Singularity Probabilities for Random Matrices over Finite Fields', *Combinatorics, Probability and Computing,* 10 (2001) 137--157.

T. Migler, K. Morrison and M. Ogle, 'How much does a Matrix of Rank k Weigh?', *Mathematics Magazine,* 79(4) (2006) 262--271.R.P. Stanley, *Enumerative combinatorics, Volume 1*, Cambridge University Press, 1997

Amelia received an AMSI Vacation Scholarship in December 2008
See: www.ice-em.org.au/vs09.html