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MATHEMATICS

The Chromatic Number of Space

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What is the minimum number of colours required to colour a metric space so that no two points of the same colour are a unit distance apart?

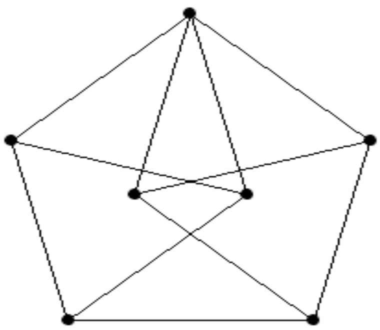
Over the 2005/2006 summer holidays I spent six weeks pondering this question, and related ones, as part of the Melbourne University's Mathematics and Statistics Department Vacation Program, funded by AMSI.

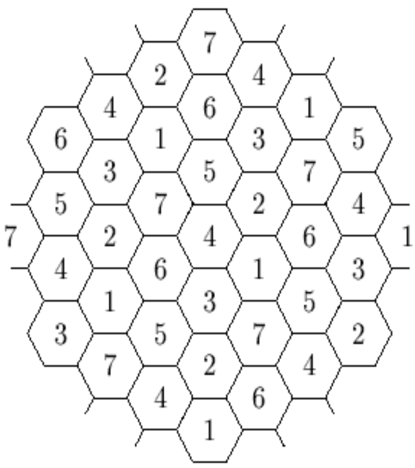
For the one-dimensional case the answer is trivial $\chi(R)=2$. However the chromatic number of R^n, $\chi(R^n)$, is not known for $n\geq2$. I spent the bulk of the period I worked looking at the two dimensional case where the bests know upper and lower bounds of 4 and 7 have been know for over fifty years.

The Moser spindle, a unit distance graph with 7 vertices, gives the lower bound as only 2 of its vertices can ever be given the same colour. The upper bound is normally shown by tiling the plane with regular hexagons of diameter slightly less than one and then colouring them is seen below.

It was a fantastic opportunity getting to read maths papers published within the last couple of years when you consider that the bulk of what we study in our undergraduate degrees is several years old.

While working on this project I also had the opportunity to go to Sydney to present a short talk on what I had been doing along with others taking part in similar programs, funded by AMSI and CSIRO, all round Australia which was a fabulous opportunity. I would strongly recommend the program to anyone considering honours in mathematics, as it is a great way to get an idea of what research in maths is like.





Jane received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarship.