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Quantification of synchronisation within coupled Lorenz systems

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Background

My project was based on the theories of chaos and synchronisation. In 1963, Edward Lorenz, a meteorologist, derived the now famous Lorenz equations from a simplified model of thermal convection [2]. The equations are renowned for both their 'chaotic' behaviour and the creation of the butterfly 'strange attractor'.

The focus of my project was to investigate the synchronisation properties of a model consisting of two coupled Lorenz systems. The coupled system is shown below:

$$\dot{X}_1 = -\sigma X_1 + \sigma Y_1$$
$$\dot{Y}_1 = -X_1 Z_1 + r_1 X_1 - Y_1$$
$$\dot{Z}_1 = X_1 Y_1 - bZ_1$$
$$\dot{X}_2 = -\sigma X_2 + \sigma Y_2 + d(X_1 - X_2)$$
$$\dot{Y}_2 = -X_2 Z_2 + r_2 X_2 - Y_2 + d(Y_1 - Y_2)$$
$$\dot{Z}_2 = X_2 Y_2 - bZ_2 + d(Z_1 - Z_2)$$

where d is the coupling strength, $\sigma=10$, $b=\frac{8}{3}$ and r_1, r_2 are control parameters.

Chaotic synchronisation implies a perfect linking of chaotic trajectories, in which they evolve in step. In a general sense, synchronisation involves the process of 'feedback' where the output of one system is given to a function of the output of the other system. In a geophysical context, the coupled Lorenz system provides a paradigm for the mutual interaction between chaotic extratropical circulation patterns in two geographical regions [1].

Research

In my research I developed a method that provides a representation of the degree of synchronisation between two coupled chaotic systems. I also investigated how the behaviour of the system depended upon the coupling strength 'd', perturbations to the initial conditions, and the time period of integration. From my results I concluded that a coupling strength of at least d=0.52 will inevitably produce perfect synchronisation. At this strength a perturbation of the initial conditions simply produces a transient synchronisation phase.

Summary

My project was both a success and a fantastic experience. It broadened my view of mathematics, enhanced my research techniques and helped develop many faculty contacts.

References

[1] A. Stefanski et al., Physica D 98 (1996) 594-598. Dynamics of coupled Lorenz systems and its geophysical implications

[2] E.H. Lorenz, J. Atmospheric Sci. 20 (1963) 130. Deterministic Nonperiodic Flow

Adam received an ICE-EM Vacation Scholarship in January 2005.
See www.ice-em.org.au/students.html#scholarship.