Unsteady Free Surface Flows

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My research project over the summer was entitled "*Unsteady Free Surface Flows*". Unsteady Free Surface Flows are a fundamentally important type of flow that exist everywhere throughout nature: from the motion of waves, to the motion of jets and even bubbles. As such, these flows are very relevant areas of research to applied mathematicians and, in particular, fluid dynamicists.

The particular problem that was addressed was that of horizontal sloshing. Put simply, this involves taking an incompressible, irrotational, invisicid fluid in a container, perturbing it, and finding out how the fluid-and in particular, the fluid surface-develops over time. The most contemporary example of horizontal sloshing would be the capsizing of the *Costa Concordia* off the coast of Tuscany, Italy. In this particular case, the movement of fluids under the deck, such as those in the partially filled fuel tanks, exacerbated the capsizing by reacting to the heave, pitch and roll of the cruise ship.

The goal of the project was to devise a simple numerical scheme which would solve the case of the aforementioned fluid in a 2D box given a small initial disturbance for the fluid that arose from a specified initial shape of the free surface on the top boundary of the fluid. Additionally, movable sides to the box and the effects of surface tension and capillary action were included. This was completed by using well known Fluid Dynamics theory to derive the equations, used to describe the fluid, from the 2D Euler equation, the hydrostatic pressure condition, the free surface condition and the incompressibility condition of the fluid. A linearized theory for small disturbances was considered but with the aim of generalizing it to the larger-amplitude case. However, as a group of partial differential equations, the resoluting nonlinear equations are too difficult to solve by hand in the general case, so they must be solved numerically.

Numerically setting up the problem involves not only the discretization of the domain-that is, breaking the continuous fluid domain into tiny grid points, but also the discretization of the aforementioned equations; using finite difference approximation schemes to approximate the partial derivatives in the equations, such that second order accuracy in space and first order accuracy in time are achieved.

Using the discretized form of the incompressibility condition, Laplace's Equation is solved numerically for the velocity potential in order to find that quantity at every point in the discretized fluid grid. Then, using this information, the discretized equations relating to fluid development over time are used to find the shape of the free surface and the velocity potential at the top of the fluid is found in the new time step. This two step process is then repeated multiple times to solve the horizontal sloshing problem.

To gauge the accuracy of the derived numerical solution, an exact solution for the simplest case of Laplace's Equation with zero Neumann conditions was subtracted from the numerical scheme with the same parameters. Upon inspection, the top of the fluid boundary exhibited wave like motion which was due to the differing periods of the oscillatory motion of the exact solution compared to the numerical solution. Further manipulating the magnitude of the time and space steps confirmed that the particular scheme derived had an error proportional to the time step and proportional to the space step squared; as expected, considering the finite difference approximation schemes used.

In summary, a numerical iterative scheme was devised for the problem of small-amplitude horizontal sloshing in a 2D box which was simple and, for reasonably valued parameters, quite stable. Additionally, the scheme provides a sturdy foundation to be built upon; it can be made more accurate, using boundary fitted coordinates and spectral methods and Chebyshev expansion, and it can be manipulated to take into account different container dimensions, even including a third dimension, and also including the fluid effects of viscosity, vorticity and compressability.

Overall, this project has been a very worthwhile and valuable experience which has given me key insight into the research process and a career in mathematics. There is no doubt that is has furthered my interest in mathematics and fluid dynamics and pursuing a career in academia. In short, it was an incredible opportunity.

Lastly, I would like to thank AMSI, CSIRO, and most importantly, my supervisor, Assoc. Prof. Michael Page, for making this opportunity possible.

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