Asset Liability Management

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This project is concerned with the methods employed to find a hedged policy, the best mix of assets in a portfolio that allows the investor to either reach a target, or meet liabilities in the future, while minimizing initial investment. A textbook by Birge and Louveaux, "*Introduction to Stochastic Programming*" familiarized the student with the basics of writing models in AMPL, while a paper by Klaassen (1998), *"Financial Asset-Pricing Theory and Stochastic Programming Models for Asset/Liability Management: A Synthesis"*, provided the framework around which this project materialized.

In finance, as with many other things in life, one cannot predict the future. When an investor buys an asset with the aim of selling it later, preferably at a higher price, he does so while exposing himself to fluctuations in the financial world, such as interest rates, foreign exchange rates, share market movements, inflation, etc. How can the investor make decisions such that regardless of how the market changes in the future (within reason, of course), his portfolio will still retain profitability? These decisions are called a hedging policy. It allows the investor to be sensitive to the uncertainty inherent in the instruments *before* he trades, so that without concrete knowledge of the situation tomorrow he may rest assured that the likelihood of a negative profit has been minimized—to his satisfaction.

This process is started with the building of a model. A discrete-state, discrete-time event tree was used to represent the different possible states of the world changing with time. The nodes represented states of the world, and the arcs the transitions through time from one state to another. This tree is described in a matrix, with a row for each node assigning a number to a node, the parent for each node, and various other data associated with a node such as the probability of an upward movement at time t to time $t+1$. With this tree in place (Leo wrote the code for this), then variables and parameters can be attached to each node; these included the amounts of each instrument to buy and sell, how much to borrow, the conditional probabilities of upward and downward movement (the tree was binomial, but this can quite easily changed), and the liabilities due to be paid at the end of each period which is also state dependent. Constraints were added to ensure that all capital is reinvested in subsequent periods, that liabilities were met, and also that holdings from one period carried over to the next.

After this the model was run to produce some results, which were analyzed. Comparing the solutions from different solvers used to derive the optimal solution showed that often these optimal solutions were not unique, i.e. even though the value for the objective function was identical for different solvers, a different mix of assets could be bought and sold to achieve the same result.

The next step is to manipulate the matrix describing the tree to remove arbitrage. Arbitrage is the ability to make money without any risk—free lunch. This arises due to the fact that 'future' data is known to the solver (that is, our model is anticipative), so the solution might be founded on these 'loopholes', which are as unpredictable as the security prices we are trying to hedge against. Hence to hinder the program from capitalizing on arbitrage opportunities (and thus unrealistically biasing the optimal solution) these must be removed while retaining all other relevant information. This involves a positive probability measure from the current period to the next such that the expected values for the one-period return for all assets are equal. Such a probability measure allows definition of prices in a backward recursive manner free of arbitrage. State and time aggregation will be implemented as the means to shave the tree of arbitrage. It also happens that doing so makes computational matters less time consuming.

The Big Day In was an enlightening experience, as I got to see the high standards of work, the research interests, and the intriguing results of the best students around the country. It was a valuable opportunity to network with many outstanding and wonderful people interested in similar fields of science. Besides, we got to practice our presentation skills; the communication of ideas and results in science is as important as the work put into the derivation thereof. The summer vacation scholarship has proved to be an educational process. I learnt how research is done in university and how researchers collaborate. I learnt how to learn, independently and actively. It has been invaluable in giving me a view of what life as a researcher is like and this will help me in deciding whether to undertake a year in Honours, and also further on, if I should be fortunate enough to pursue graduate studies. I would like to thank my supervisors Dr Leo Lopes for putting up with me and pushing me on, and also for the support of Dr Kais Hamza throughout the summer. My gratitude also goes to AMSI and CSIRO for their generous financial support and also for organizing Big Day In.

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