The Mixture Exponential Model

Jason Tran
The University of Sydney

The mixture exponential model i.e. (1-p)exp(-x) + p/μ exp(-x/μ) is of interest to study because it occurs in neural biology. One such example is that within the membrane of our cells we have ion channels, which allows the flow of calcium ions. It has been said that these ion channels have three states: "Open 1", "Open 2", "Closed" whereby the transition between each of these states is modelled by a continuous time Markov chain. The detection used however cannot distinguish between states "Open 1" and "Open 2", and hence we create an aggregate Markov Model where we do not distinguish between the two open states, and thus the waiting time in a single "Open" state is then modelled by a mixture exponential distribution.

One may also think of the mixture exponential model as a contamination model where most of the data comes from an exponential distribution of mean 1, where there is a slight contamination in the data in which a small proportion comes from an exponential distribution of mean μ.

Suppose we have n observations that are independently and identically distributed with some regular parametric density. Often in hypothesis testing we use the Log-Likelihood ratio since it has an approximate χ2 distribution. This however only works when certain regularity conditions hold, and in our case of the mixture exponential model these conditions are not met and hence we cannot use the well known convergence in distribution of the Log-Likelihood Ratio. What we attempt to do then is find the distribution of the Log-Likelihood for testing whether data come from a single exponential model of mean 1 against whether the data come from a mixture exponential model.

Liu et al. (2003) had showed that there was a limiting distribution for the case where the model has a "contamination" counterpart with mean less than two, where it was said that the Log-Likelihood ratio was bounded in probably by loglog(n). Liu also showed that this ratio converged in distribution. Through the use of non-parametric convergence results and some Wong and Shen Theory (2001) we believe that if the "contamination" mean was greater than two then the Log-Likelihood ratio was bounded in probability by logloglog(n). This would ultimately lead to the conclusion that often Liu's convergent distribution would be enough to test the above hypothesis (if n was large enough)

This Vacation Scholarship has allowed me to explore different approximation techniques as various integrals that were considered did not have closed forms. Often technology such as Taylor Approximations as both linear and quadratic orders were helpful as well as Babylonian methods in approximation square roots. The notion of Entropy was employed liberally as a method of attacking this problem.

This experience has allowed me to explore my own independent mathematical ideas and move beyond the boundaries of any set course work. As such my intuition and abstract thought have matured greatly through the process. The ability to communicate with other enthusiasts has allowed for constructive brain-storming as well as a sense of community and achievement when a problem has been solved.

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Postal Address: 111 Barry Street
c/- The University of Melbourne
Victoria 3010 Australia

Email: enquiries@amsi.org.au
Phone: +61 3 8344 1777
Fax: +61 3 9349 4106
Web: www.amsi.org.au