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The diffusive pair contact process

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I took part in the 2005/2006 Summer Vacation Scholarship Program, which allowed me to work on a research project at the University of Melbourne, Department of Mathematics and Statistic, under the supervision of Dr. Iwan Jensen, for a period of 6 weeks. This was a great opportunity as I plan on going on to Honours, and this project gave me a real feel for what that might be like.

My project involved classifying a particular model of percolation, called the pair contact process with diffusion. In Statistical Mechanics, there are two types of systems, equilibrium systems, and non-equilibrium systems. We deal with equilibrium systems everyday, like when two objects at different temperatures transfer heat with each other until they are both the same temperature. The two objects approach a single equilibrium no matter where they both start. Non-equilibrium systems don't do that. They can get stuck in states.

In the pair contact process (PCP), for instance, a one dimensional lattice is filled with particles. Two processes, fission and annihilation, are randomly applied with probability p and 1-p respectively. If the probability of fission is low, the pairs will annihilate, and there will be no particles left to react. This is called an absorbing state. If the probability of fission is high, particles will be created far more often, and the lattice will never be empty and will remain in what is called an active state.

In the case of a low value for p, some particles can become 'stranded' in that the particles are left without a partner. If the particles are allowed to diffuse in the lattice as well, then they can't get stuck. This is the case of the Pair Contact Process with Diffusion (PCPD), the model that I spent my time examining.

To do this I wrote a program that simulated two particles being placed in to the middle of a lattice. The particles were then allowed to diffuse with probability D, undergo fission with probability (1-D)p, or annihilate with a probability of (1-D)(1-p). This was continued on a turn-based approach, where one particle was chosen at random to perform an action, the action was performed if it was possible, and then the time was updated by 1/n where n was the number of particles at that point in time. The time was updated like this because one time unit corresponded to a complete update of the lattice. The lattice was updated until there was only one or no particles left in the lattice, and no more pairs could be formed, or until the time reached 10000 units.

I did this several times while varying the diffusion and fission rates to observe some of the features of the model, such as the survival probability (the probability of the system being active after t time) the particle density, and the pair density. I used these to find significant characteristics of the PCPD, such as the critical probability, where the system stays in a state between an active state and an absorbing state.

T'Mir received an ICE-EM Vacation Scholarship in December 2005.
See www.ice-em.org.au/students.html#scholarships05