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TIME DELAYS INVOLVED IN THE ACCOUSTIC INTERACTIONS BETWEEN BUBBLES

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Work on the time delays involved in the acoustic interactions between bubbles. Bubbles in nature exist in varies sizes and shapes. They usually exist in a system of bubbles and are found to be oscillating in different modes. In this review we only examine on linear modes established for a short period of time on acoustic excitation. Behaviour of a single, isolated bubble is used as a logical starting point, though in nature bubbles very rarely exist in perfect isolation. The self-consistent theory is embraced and is used to develop mathematical models for the acoustic interactions on a bubble in a system of bubbles. Examining this model, we review the time delays involved the acoustic interaction between a bubble and a neighbouring bubble.

MOTIVATION

Like mentioned above bubbles exist in a wide range of sizes and different shapes. In this section we discuss the applications of the bubbles in practical situations. The motivation to this work is to explore the new techniques and approaches that can lead to new applications or improve the existing applications.

Ordering the applications based on the size of the bubbles not only seems to be a sensible approach but also gives us the idea of the wide range of bubbles that exist. The resonant sound produced by gas bubbles in volcanic magma which are 100m in size, is being used to predict the seriousness of the eruption that's about to take place.[2-4] Methane bubbles that are arise from the ocean bottoms have been a subject of research since a considerable number of years. These bubbles usually exist in millimetres.[5-6] Industries such as chemical and process, have been playing a vital role as the reason for the studies and research of bubbles and their frequencies.[7-12]

The field of medicine is one of the most important example of where bubbles need to be handled with extreme care. If a bubble that is a tenth of millimetre in size enters arteries during a surgery, they can cause serious damages.[13] Bubbles these sizes are detected and removed acoustically.[14]

Bubbles in the sizes of microns have been used be in diagnosing and healing patients with illnesses such as stroke[15-17] and being used in drug delivery.[18-21] The above-mentioned applications can give us a clear idea of the sizes of bubbles that are being used in practical situations. Theoretically, bubbles are believed to exist in sub micrometre and nanometre size and can even be used to generate nuclear fusion.[22-23] This widens the range of the size of the bubbles over nine orders of magnitude. This leaves us with enough motivation to explore more about the existence and behaviours of bubbles and use them practically.

BACKGROUND AND HISTORY

Works on bubbles and their behaviours dates to the years of Rayleigh[24] who analysed the dynamics of a single collapsing. Minnaert[43] then came up with the natural frequency of a bubble which paved the way to extensive research and studies on the behaviour of bubbles. Theories were developed to predict the frequencies of a system of bubbles.[25-32] Strasberg[33] analysed the effect of a rigid boundary next to a bubble and came up with the concept of the mirror image. This concept allows us to treat the rigid boundary that is right next to an oscillating bubble as a mirror image of the bubble itself. The image concept does have its own limitations when the bubbles are large and have considerable contact with a rigid boundary.[1]

APPROACH

It is important to acknowledge the approach this work as used to develop the theories and achieve its goals. The citied applications of bubbles and its behaviours that were discussed in the 'Motivation' section imply that the when the size of single bubble increases its natural frequency decreases. Though this will be an acceptable starting point, we also discussed that bubbles rarely exist in isolation and mostly found in groups.

We limit ourselves and assume that the bubbles remain spherical and have a small oscillation as this validates the linear theory. Since our review is about the interaction between bubbles we consider the bubbles to be ‘naked’ – which implies that they don’t have any artificial shell coatings. These artificial shell coatings are found in bubbles that are used in the field of medicine and they certainly do modify the frequencies of the bubbles.[34-37] The other assumption we make is that the centres of bubbles interacting are stationery relative to each other which helps us to exclude the occurrence like Bjerknes forces.[38]

THEORIES AND CONCEPTS

Fundamentals

Rayleigh-Plesset equation is one of the very first mathematical models on bubbles and was derived by applying the equations of mass conservation and momentum balance to the liquid around the bubble and the ideal gas law for the pressure at the bubble wall.[1]

$$\ddot{R}R + \frac{3}{2}R^2 = \frac{1}{\rho}\left[\left(P_0 + \frac{2\sigma}{R} - p_s\right) - \left(P_0 + \frac{2\sigma}{R} - p_v\right)\left(\frac{R_0}{R}\right)^{3k} + \frac{4\mu}{R}\dot{R}\right] \quad (1)$$

$R(t)$	Instantaneous radius of the bubble	p_v	Vapour pressure of the liquid
R_0	Bubble’s equilibrium radius	σ	Surface tension
ρ	Liquid density	k	Polytrophic exponent
μ	Viscosity of the liquid		

It is clear that (1) is non-linear and suggests extreme changes in behaviour if the radius gets small.

Practically this non-linearity is used in applications and technologies. However, for our analysis it is useful if there’s linearity maintained. So, we assume that the change in radius ($R(t)$ - R_0) is smaller when compared to R_0 and introduce the term δ which is the change in radius. Using $\delta(t) = R(t) - R_0$ we linearize (1) and arrive at (2) which is a harmonic motion equation. We ignore the vapour pressure and surface tension to keep it simple.

$$\ddot{\delta} + b\dot{\delta} + {\omega_0}^2\delta = 0 \quad (2)$$

b	Damping term	ω_0	Natural frequency of the bubble

The key conclusion we arrive at the relation between the natural frequency of the bubble oscillation and its size. (3)

$$\omega_0 = \left(\frac{3kP_0}{\rho}\right)^{\frac{1}{2}} \frac{1}{R_0} \quad (3)$$

Minnaert arrived at (3) by equating kinetic and potential energy of the oscillation. (3) is very significant since it allows us to estimate the natural frequency of bubbles if given the size even though it was derived after a few simplifications and assumptions.

Coupled-oscillator theory under self-consistent approach

So far, we have looked at the works and analysis done on a single bubble that is osculating on its own. Now we turn our attention to a system of bubbles and how a bubble's oscillation is affected by the neighbouring bubbles in the system. You will find that this will slowly develop into our area of interest which is how time delays are involved in acoustic interactions between bubbles.

Works from the past[36,25,26,28-32] have successfully represented the effect of other bubbles on the bubble of our interest (ith bubble) by adding terms to the right-hand side of (2).

$$\ddot{\delta}_i + b_i\dot{\delta}_i + {\omega_{0i}}^2\delta_i = -p_T \quad (4)$$

Here p_T is an infinite sum of a function of R_{0j}, s_{ji} and the distance between the ith and the jth bubble. We get the negative sign due to the pressure disturbance the neighbouring bubble has on the affected bubble would lead to a reduction in size. Also, do recall that the distance between the bubbles s_{ji} is assumed to be constant with time. Another assumption that is made here is the source of the acoustic field that is affecting the bubble is monopolar.[1] This restricts (4) being used only when $s_{ji} >> R_{0j}$.

The reason behind the infinite sum on the right-hand side of (4) is discussed below. The total pressure disturbance on the i^th bubble is the sum of pressure disturbance that is due to each of the neighbouring bubbles in the system. But, the vibration released by a bubble once it reaches a second bubble gets reflected to the one that emitted it which creates an infinite series. Though the number of bubbles is finite it is evident that determining the number is practically not an easy task.

To solve this issue, out of the many approaches available, we use an approach called the 'self-consistent approach' which was used by Tolstoy[26] not only in acoustics but also in similar situations that can be found in the world of physics.

As much as the self-consistent approach simplifies our issue, by using it we lose the physical meaning of the dependent variable we have in (4).[1] So we introduce a 'self-consistent dependent variable ($x_i(t)$) to stress the difference from the physical dependent variable ($\delta(t)$).

$$\ddot{x}_i + b_i\dot{x}_i + {\omega_0}^2 x_i = -\sum_{j=1\, j\neq i}^{N} \frac{R_{0j}}{s_{ji}}\ddot{x}_j \quad (5)$$

Symmetric and antisymmetric modes

Let's now take a look at a system that consists only two bubbles in it. In this case (5) can be rewritten to our specific system as follows.

$$\ddot{x}_1 + b_1\dot{x}_1 + {\omega_0}^2 x_1 = -\frac{R_0}{s}\ddot{x}_2 \quad (6)$$

$$\ddot{x}_2 + b_2\dot{x}_2 + {\omega_0}^2 x_2 = -\frac{R_0}{s}\ddot{x}_1 \quad (7)$$

To keep things simple and not necessarily, we assume the equilibrium radius of both the bubbles are equal and R_0 and the bubbles contain the same gas and the same pressure which also means they have the same natural frequency ω_0 and damping term b.

By manipulating the equations, we arrive at symmetric (by adding the equations and solving) and antisymmetric (by subtracting the equations and solving) modes. In symmetric mode where the bubbles oscillate in phase we find the natural frequency ω_+ and damping term b_+ to be as follows.

$$\omega_+ = \frac{1}{(1+R_0/s)^{1/2}}\omega_0 \quad (8)$$

$$b_+ = \frac{b}{(1+R_0/s)} \quad (9)$$

In antisymmetric mode where the bubbles oscillate in antiphase it has a natural frequency of ω_- and damping term of b_-.

$$\omega_- = \frac{1}{(1-R_0/s)^{1/2}}\omega_0 \quad (10)$$

$$b_- = \frac{b}{(1-R_0/s)} \quad (11)$$

INTRODUCING TIME DELAYS

Equation (4) and its self-consistent version (5) assume that the liquid is incompressible which also means that the affected bubble instantly feels the pressure field of a neighbouring bubble. However, sound does have a finite speed in a liquid which are compressible. This leads to a time delay in the oscillation of the neighbouring bubble being felt by the affected bubble. This issue has been dealt by previous researchers[39-42] with the ansatz of time delaying the couple terms.[1]

$$\ddot{x_i}(t) + b\dot{x_i}(t) + {\omega_0}^2 x_i(t) = -\sum_{j=1\ j\neq i}^{N} \frac{R_{0j}}{s_{ji}}\ddot{x}_j\left(t-\frac{s_0}{c}\right) \quad (11)$$

Here c is the speed of the sound in the liquid. Recall that the it has been already assumed that the source of the vibration is monopolar and to be valid for s_{ji} >> R_{0j}. In addition to that consider the cases

1. $\frac{s_0}{c} << \frac{1}{f}$ (time delay is much smaller than the natural period of the oscillation)
2. $s_0 >> R_0$ (the bubbles are much smaller than the distance between them)

This allows us to do a Taylor series expansion of the time delay terms and arrive at

$$\frac{R_0}{s_{ij}}\ddot{x}_j\left(t - \frac{s_{ij}}{c}\right) \cong \left(\frac{R_0}{s_{ij}}\ddot{x}_j + \ \omega_0{}^2\frac{R_0}{c}\dot{x}_j\right) \quad (12)$$

$$\ddot{x_i}(t) + b\dot{x_i}(t) + \omega_0{}^2 x_i(t) = -\sum_{j=1\, j\neq i}^{N}\left(\frac{R_0}{s_{ij}}\ddot{x}_j + \ \omega_0{}^2\frac{R_0}{c}\dot{x}_j\right) \quad (13)$$

Stability of differential equations

Theory

The stability of equilibrium of a differential equation is discussed below from its basics. To begin with the fundamentals, let's analyse the stability of the below given equation.

$$\frac{dx}{dt} = f(x,t)$$

The stability of the equilibrium is analysed by finding the equilibrium point i.e. $\frac{dx}{dt} = f(x,t) = 0$. Assume that x_e is a point where equilibrium is found which also implements $f(x_e) = 0$. Let's look at the simplest of cases which is a linear one: $\frac{dx}{dt} = ax$(where 'a' is a parameter).

So, the equilibrium point for the above case is $x_e = 0$. The solution for the above function is given as $x(t) = ce^{at}$ which indicates that the stability depends on a.

- If a < 0: It's an exponential decay and the equilibrium is stable
- If a > 0: It's an exponential growth and the equilibrium is unstable

Now let's look at a case where $f(x)$ is non-linear. In this case at any given point of equilibrium what determines the stability would be the tangent line drawn at the point of equilibrium which is $f'(x_e)$

- If $f'(x_e) < 0$: It's an exponential decay and the equilibrium is stable
- If $f'(x_e) > 0$: It's an exponential growth and the equilibrium is unstable

The case where $f(x) = a$ can also be treated with this approach in which $f'^{(x_e)} = a$.
Now let's look at an approach for a much more complicated case.

$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = 0 \quad (14)$$

We introduce $y = \frac{dx}{dt}$.

$$\therefore \frac{d^2x}{dt^2} = \frac{dy}{dt}$$

Applying what we have introduced and attained to (14) will give us

$$\frac{dy}{dt} = -\frac{(a_1 y + a_0 x)}{a_2}$$

We now have a system ODE at our hands.

$$\frac{dx}{dt} = y = f(x,y)$$

$$\frac{dy}{dt} = -\frac{(a_1 y + a_0 x)}{a_2} = g(x,y)$$

An equilibrium point is a point E (x^*, y^*)satisfying the following conditions

$$\frac{dx}{dt} = f(x^*, y^*) = 0$$

$$\frac{dy}{dt} = g(x^*, y^*) = 0$$

Now we analyse the linear stability of this equilibrium point. To start with we introduce U and V.

$$U = x - x^*$$

$$V = y - y^*$$

Doing further maths on this

$$\frac{dU}{dt} = \frac{d(\,x - x^{*})}{dt} = \frac{dx}{dt} = \,f(x,y)$$

Applying the Taylor Series expansion at (x^{*}, y^{*}) (U = 0, V= 0) we arrive at,

$$\frac{dU}{dt} = \frac{\partial f(x^{*},y^{*})}{\partial x}(\,x - x^{*}) + \frac{\partial f(x^{*},y^{*})}{\partial y}(\,y - y^{*})$$

$$\frac{dU}{dt} = \frac{\partial f(x^{*},y^{*})}{\partial x}U + \frac{\partial f(x^{*},y^{*})}{\partial y}V$$

Similarly, we can arrive at,

$$\frac{dV}{dt} = \frac{\partial g(x^{*},y^{*})}{\partial x}(\,x - x^{*}) + \frac{\partial f(x^{*},y^{*})}{\partial y}(\,y - y^{*})$$

$$\frac{dV}{dt} = \frac{\partial g(x^{*},y^{*})}{\partial x}U + \frac{\partial f(x^{*},y^{*})}{\partial y}V$$

Now we have another system of ODE,

$$\frac{dU}{dt} = a_1 U + \, a_2 V$$

$$\frac{dV}{dt} = a_3 U + \, a_4 V$$

From the we above equations we aim to arrive at a characteristic equation that will help us learn the stability of the equilibrium.

Assume the above system has a solution in the form as below.

$$\binom{U}{V} = \binom{c_1}{c_2} e^{\lambda t}$$

c_1 and c_2 are constants that cannot be 0 simultaneously.

$$U \;=\; c_1 e^{\lambda t}$$

$$V \;=\; c_2 e^{\lambda t}$$

Substituting the above in the system and then by simplifying we arrive at,

$$\lambda c_1 \;=\; a_1 c_1 + \, a_2 c_2$$

$$\lambda c_2 \;=\; a_3 c_1 + \, a_4 c_2$$

Here,

$$a_1 = \frac{\partial f(x^*, y^*)}{\partial x}$$
$$a_2 = \frac{\partial f(x^*, y^*)}{\partial y}$$
$$a_3 = \frac{\partial g(x^*, y^*)}{\partial x}$$
$$a_4 = \frac{\partial g(x^*, y^*)}{\partial y}$$

We now convert the simultaneous equations into matrix form,

$$\lambda \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

We name matrix $\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$ as A and matrix $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ as C and rearrange the above equation

$(\lambda I - A)C = 0$ (here 'I' is a 2x2 identity matrix)

Since C is a non-zero vector $det(\lambda I - A)$ must be equal to zero.

$$det(\lambda I - A) = de\,t\left(\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \right) = det \begin{pmatrix} \lambda - a_1 & -a_2 \\ -a_3 & \lambda - a_4 \end{pmatrix} = 0$$

$$\lambda^2 - (a_4 + a_1)\lambda + \det(A) = 0$$

Analysing the roots of the above characteristic equation will let us the decide the stability of the equilibrium.

Application of Theory to our case

Let's consider the simplest of systems which is a system of two bubbles acoustically interacting. To keep things simple and not necessarily we assume both the bubbles have an equilibrium radius of R_0 and contain the same gas inside them which also implements that their damping constant b and natural frequency ω_0 are the same. Now we apply (13) to our system and arrive at the following equations,

$$\ddot{x_1} + b\dot{x_1} + {\omega_0}^2 x_1 = -(\frac{R_0}{s_{ij}}\ddot{x_2} + {\omega_0}^2\frac{R_0}{c}\dot{x_2})$$

$$\ddot{x_2} + b\dot{x_2} + {\omega_0}^2 x_2 = -(\frac{R_0}{s_{ij}}\ddot{x_1} + {\omega_0}^2\frac{R_0}{c}\dot{x_1})$$

We assume the above system has a solution in the form as below,

$$x_1 = c_1 e^{\lambda t}$$

$$x_2 = c_2 e^{\lambda t}$$

Applying the equation to the system and simplifying and rearranging we arrive at,

$$(\lambda^2 + b\lambda + {\omega_0}^2)c_1 + (\frac{R_0}{s}\lambda^2 + {\omega_0}^2\frac{R_0}{c}\lambda)c_2 = 0$$

$$(\lambda^2 + b\lambda + {\omega_0}^2)c_2 + (\frac{R_0}{s}\lambda^2 + {\omega_0}^2\frac{R_0}{c}\lambda)c_1 = 0$$

The above two equations are linear in terms of c_1 and c_2..

$$mc_1 + nc_2 = 0$$

$$mc_2 + nc_1 = 0$$

Here $m = (\lambda^2 + b\lambda + {\omega_0}^2)$ and $n = (\frac{R_0}{s}\lambda^2 + {\omega_0}^2\frac{R_0}{c}\lambda)$. When we try to solve the equations for c_1 and c_2 we arrive at

$$(m^2 - n^2)c_1 = 0$$

$$m = n$$

$$(\lambda^2 + b\lambda + {\omega_0}^2) = (\frac{R_0}{s}\lambda^2 + {\omega_0}^2\frac{R_0}{c}\lambda)$$

$$\left(1 - \frac{R_0}{s}\right)\lambda^2 + \left(b - {\omega_0}^2\frac{R_0}{c}\right)\lambda + .{\omega_0}^2 = 0$$

By analysing the roots of the above quadradic equation in terms of λ will determine the stability of the equilibrium of our system.

$$\lambda = \frac{-\left(b - {\omega_0}^2\frac{R_0}{c}\right) \pm \sqrt{\left(b - {\omega_0}^2\frac{R_0}{c}\right)^2 - 4\left(1 - \frac{R_0}{s}\right){\omega_0}^2}}{2\left(1 - \frac{R_0}{s}\right)}$$

- If Re(λ) < 0: It's an exponential decay and the equilibrium is stable
- If Re(λ) > 0: It's an exponential growth and the equilibrium is unstable

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