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The Effect of Population Heterogeneity on the Spread of Contagious Diseases

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Contents

1 Abstract

In this report, population heterogeneity refers to the idea that di erences in individual choices among groups of individuals result in di erences in outcomes. In the context of a pandemic, individuals can either choose to obey or disobey government interventions that aim to reduce the spread of disease. A greater degree of obedience towards interventions is predicted to lead to a bene cial outcome for the population. A lower degree of obedience towards interventions is predicted to lead to a detrimental outcome for the population. The aim of this project is to examine an epidemiological model that simulates the transmission of COVID-19 among homogeneous and heterogeneous populations, where both the e ectiveness of di erent interventions and population heterogeneity are considered.

2 Introduction

The COVID-19 pandemic, caused by a highly infectious respiratory disease, has shed a spotlight on the use of the SIR model. This model provides us with the ability to predict the transition of individuals between susceptible, infectious, and recovered states. A population is generally heterogeneous, meaning there exists di erent groups of individuals that make di erent choices. In relation to the COVID-19 pandemic, civilians may be informed by the government to self-isolate, practise proper hand hygiene, and to wear masks in order to prevent the transmission of disease. However, an increase in the number of individuals who misjudge the threats posed by COVID-19 can occur.

Countries or cultures with tight social norms, where the actions of individuals adhere closely to societal expectations, are much more likely to abide by COVID-19 interventions than those with loose social norms, where the actions of individuals tend to deviate from societal expectations more often [8]. These di erences in attitudes are a major cause of population heterogeneity. The simulated population will be split into two opposing groups. Individuals from group one do not follow government interventions. Individuals from group two follow government interventions.

Simulations of mathematical models are investigated to measure changes in four particular quantities within both homogeneous and heterogeneous quantities depending on the day that interventions are introduced as well as the proportion of individuals in group one and group two. These four quantities are the maximum number of infections (I_{max}), time to maximum infections (t_{max}), eradication time or when there is less than one infected individual in the population (t_{ext}), and the total number of recovered individuals (R_1).

3 Statement of Authorship

The workload was divided as follows:

- Tianze Wei adapted and built on existing MATLAB code, located and investigated references for the report, produced mathematical results in the form of graphs and tables, reported and interpreted the results, and typed up the report.
- Mark Nelson provided the basis for the MATLAB code, supplied the theoretical resources, assisted with the numerical simulations, and proofread the nal report.

4 Applications of the SIR Model for a One-Group Homogeneous Population

An SIR epidemiological model predicts the number of individuals infected by a contagious disease within a closed population of a constant size. This model, as represented graphically in Figure 1, divides the population into three categories and explores the spread of the disease through the use of di erential equations which model the transition of individuals between these categories [1]. Susceptible individuals are denoted by S. Infectious individuals are denoted by I. Recovered individuals are denoted by R. These groups can be thought of as compartments which individuals can enter and leave, but only from S to I and then from I to R. At every point in time, the sum of susceptible, infectious, and recovered individuals is constant and equal to the total population.

Figure 1: The transition of susceptible, infectious, and recovered individuals in the SIR one-group homogeneous model.

The three differential equations that are used for the SIR model are:

$$\frac{dS}{dt} = \quad \beta \quad I \quad \frac{S}{N}, \tag{1}$$

$$\frac{dI}{dt} = \beta \quad I \quad \frac{S}{N} \quad b \quad I, \tag{2}$$

$$\frac{dR}{dt} = b \quad I. \tag{3}$$

The blue components represent the infection rate and the red components represent the recovery rate. The transmission rate of the disease or the expected number of individuals that an infected person can be expected to infect each day is represented by β. The average infectious period of an individual before they become recovered is represented by 1/b. The basic reproduction number, which is the number of people that an infectious individual infects before recovery, is represented by R_0. Equation 4 depicts the relationship between these parameters.

$$R_0 = \frac{\beta}{b}. \tag{4}$$

At the beginning of a pandemic barely any individuals are initially infected so nearly everyone is a susceptible individual, resulting in the approximation $S \quad N$ and $\frac{S}{N} \quad 1$. This approximation is then substituted into the equation $\frac{dI}{dt} = b(R_0 \quad \frac{S}{N} \quad 1) \quad I$. When $R_0 < 1$, the disease dies out eventually as $\frac{dI}{dt} < 0$. When $R_0 > 1$, the disease self propagates and exponentially grows in the population as $\frac{dI}{dt} > 0$.

5 Modelling Effect of Intervention Methods

The effective infectious contact or transmission rate due to intervention i can be expressed as $\beta_i = \beta \quad (1 \quad m_i)$. The effective basic reproduction number due to intervention i can be expressed as $R_i = R_0 \quad (1 \quad m_i)$.

5.1 Level One Social Distancing

Social distancing reduces the transmission rate in a manner that can be hard to quantify. This is because the level of restrictions that social distancing brings about varies greatly. Social distancing can refer to the maximum number of individuals allowed in a certain area, stay at home orders that confine individuals completely in their homes, a minimum separation distance between non-family members, and the closure of non-essential businesses.

With regards to the COVID-19 incidence in the U.S, analysis was collected on the effect of social distancing policies at a county level [7]. The restrictive level of social distancing precautions in each U.S county was valuated from `One' being the lowest restrictive level to `Five' being the highest restrictive level. Each one unit incremental increase was associated with a 29% reduction in the incidence of COVID-19, with the range of the reduction being 13-43%. In this report, it is estimated that social distancing will result in a 30% reduction in the incidence of COVID-19 [4]. This incidence reduction value now needs to be translated into a reduction value in the transmission rate of the disease.

We first define incidence as the number of new infections that occur per day. This is denoted by

$$\frac{dI}{dt} = \beta \cdot I \cdot \frac{S}{N} - b \cdot I.$$

If we then denote t as the number of days elapsed since the initial infection within a population and N as the total population, the derivative can be approximated as

$$\frac{dI}{dt} = \frac{I(t+h - I(t))}{h}.$$

Here, h is the step size. Suppose that $h = 1$, yielding

$$\frac{dI}{dt} = I(t+1) - I(t).$$

This is a representation of the number of new cases on day t. Let M denote the number of new cases each day. Using what we established earlier,

$$M = I(t) \cdot (\beta \cdot \frac{S}{N} - b).$$

Assuming that $S \approx N$ as susceptible individuals make up nearly all of the population when the initial infection starts, then

$$M = I(t) \cdot (\beta - b).$$

Assuming that decreasing the value for β to $\beta \cdot a$, where $a < 1$, the number of new cases changes from M to $c \cdot M$, where c is the reduced value for incidence rates and $c < 1$. We then derive the following calculations:

$$M = I(t) \cdot (\beta - b),$$

$$cM = I(t)(a \cdot \beta - b),$$

$\Longrightarrow$

$$\frac{1}{c} = \frac{\beta - b}{a \cdot \beta - b},$$

$\Longrightarrow$

$$a \cdot \beta - b = c \cdot (\beta - b),$$

Since

$$R_0 = \frac{\beta}{b},$$

$\Longrightarrow$

$$\frac{1}{R_0} = \frac{b}{\beta},$$

$\Longrightarrow$

$$a = \frac{b}{\beta} + c \cdot (1 - \frac{b}{\beta}) = \frac{1}{R_0} + c \cdot (1 - \frac{1}{R_0}).$$

Substituting the parameter value of $R_0 = 2.79$ [3] and $c = 0.7$ (as we are assuming that social distancing results in a 30% reduction in COVID-19 incidence rates) into this last equation, we obtain $a = 1/2.79 + 0.7 \times (1 - 1/2.79) \approx 0.80$. This means that social distancing results in a 20% transmission reduction rate for COVID-19. Hence, $m_{\text{social}} = 0.2$ and $\beta_{\text{social}} = 0.8 \times \beta$.

5.2 Proper Hand Hygiene

Enforcement of proper hand hygiene is estimated to reduce the transmission rate by 24-31% [4, 9]. The reduction here is estimated to be 30%. Hence, $m_{\text{hygiene}} = 0.3$ and $\beta_{\text{hygiene}} = 0.7 \times \beta$.

5.3 Mask Enforcement

Enforcement of cotton masks reduces both the inward transmission of the disease to a susceptible individual and the outward transmission of disease virus by an infectious individual. Reductions in the transmission rate are estimated to be 50% for outward transmission and 20% for inward transmission [4]. If everyone wears a cotton mask, then the transmission rate is reduced to 40% of its original value [5, 6]. Hence, $m_{\text{mask}} = 0.6$ and $\beta_{\text{mask}} = 0.4 \times \beta$.

5.4 All Three Interventions

The beta reduction value for a combination of all three interventions can be calculated through finding the intersection of their respective reduction rates: $\beta_{\text{all}} = 1 - (1 - 0.2) \times (1 - 0.3) \times (1 - 0.6) = 0.776$, resulting in a 77.6% reduction in transmission rate of the disease. This assumes that the effectiveness of all three interventions are independent of each other. Hence, $m_{\text{all}} = 0.776$ and $\beta_{\text{all}} = 0.224 \times \beta$.

6 Results for the One-Group Homogeneous Population

In a one-group homogeneous model, all individuals either follow COVID-19 restrictions or they do not follow COVID-19 restrictions.

6.1 Initial Conditions and Equations

For the one-group model, the initial necessary parameters are the total population $N = 3000$, the per-capita recovery rate $b = 1/14$ days^{-1}, the basic reproduction number $R_0 = 2.79$ [3], and the rate of transmission among the population $\beta = R_0 \times b = 2.79 \times 1/14 \approx 0.20$. The initial number of infectious individuals in the population, denoted by I(0), is within the range of $1 \leq I(0) \leq 5$.

A period of one year was simulated from Day 0 to Day 364. MATLAB was programmed to return integration points for each day ($t = 0, 1, 2, ..., 364$). When calculating the maximum number of infected individuals, the value is rounded down to yield an integer number. When calculating the total number of recovered individuals, the value is rounded up to provide an integer number. It is assumed that the total number of current infected individuals on the nal integration point rounds down to zero and that the integration period is su ciently long enough for this to occur.

6.2 Summary of Results

Earlier government interventions decrease the maximum number of infections (I_{max}) and the total number of recovered individuals (R_1). Contrarily, earlier government interventions increase eradication time (t_{ext}). This means that the disease burns through the population at a faster rate at the expense of being eradicated more quickly. If the basic reproduction number is greater than one when an intervention is introduced, the time to maximum infectives (t_{max}) either decreases to a minimum point before increasing and plateauing out, or decreases before plateauing out immediately, the later the intervention. If the basic reproduction number is less than one when an intervention is introduced, t_{max} simply increases before plateauing out, the later the intervention. Supporting gures and tables for the one-group homogeneous model are placed in Appendix A.

6.2.1 Effect of No Intervention ($m_i = 0$)

Table 1 depicts that as the number of initial infectives in the population increases, the number of maximum infections increase. Contrarily, the time to maximum infections and the eradication time of the disease decreases the quicker the eradication time. The total number of recovered individuals remains the same.

I(0)	I_{max}	t_{max}	t_{ext}	R_1
1	821	68	202	2773
2	822	62	196	2773
3	822	59	193	2773
4	822	57	191	2773
5	823	55	189	2773

Table 1: Di erences in quantities as a function of the initial number of infectious individuals for the one-group population with no interventions.

To calculate the total number of recovered individuals at any particular time, both sides of the equation $\frac{dR}{dt} = b\,I$ need to be integrated with respect to t:

$$\int \frac{dR}{dt}\,dt = b\int I(t)dt.$$

Now, a dummy variable τ needs to be introduced for integration purposes. At $t = 0$, $R = R_i$ and at $t = \tau$, $R = R(\tau)$, where R represents the number of recovered individuals. This means that the following upper and lower bounds can be introduced into the integration equation:

$$\int_{R_i}^{R(t)} \frac{dR}{d\tau} \; d\tau = b \int_0^t I(\tau)d\tau.$$

After integrating the LHS of the equation, we obtain:

$$R(t) = R_i + b \int_0^t I(\tau)d\tau.$$

We assume that $R_i \geq 0$ for the initial number of recovered individuals and that $I(t) \geq 0$ as the number of infectives in the population will always be greater or equal to zero at any point in time. This yields the expression $R(t) \geq R_i \geq 0$. The total number of recovered individuals cannot exceed the total population N. This means that:

$$R(t) = R_i + b \int_0^t I(\tau)d\tau \leq N.$$

As $0 \leq R_i \leq N$, it can be concluded that $\lim_{t \to \infty} I(\tau) = 0$. Here, the final expression conveys that over the course of the pandemic, the number of infectives will decrease to zero and the number of total recovered individuals will increase and approach the bounded value for the population size $N = 3000$.

Figure 2 illustrates that the more initial infectives there are in the population, the lower the time to maximum infections and the lower the eradication time. This means that the transition of infectives is spread over a shorter period of time, which is detrimental on a larger scale of initial infectives as the healthcare system can easily be flooded with patients.

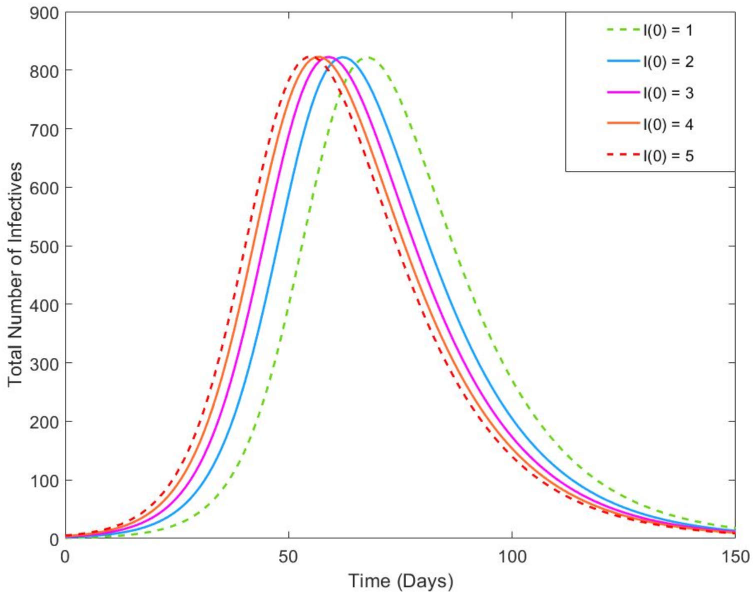

Figure 2: The total number of individuals infected in a population as a function of the initial number of infected individuals.

6.2.2 Effect of All Three Interventions (m_i = 0.776)

When all three interventions are introduced into the population on a particular day (t_{all}(days)), a reduction in the basic reproduction number R_{all} = (1 0.776) R_0 = 0.62 occurs. The earlier the introduction of interventions into the population, the smaller the quantities in Table 2. The maximum number of infectives increases and plateaus the later the interventions are introduced as by Day 60. This is because the disease has likely infected most members of the population already, meaning that any further interventions will not save any more susceptible individuals from being infected. In Figure 3, the green dotted curve shows an especially bene cial outcome due to the extremely low size for maximum number of infectives and total recovered individuals as the spread of disease is over almost instantly when the intervention is introduced, especially since $R_0 < 1$.

t_{all}(days)	I_{max}	t_{max}	t_{ext}	R_1
20	61	20	162	183
40	489	40	182	1187
60	823	55	170	2327
80	823	55	170	2659
100	823	55	174	2740
1	823	55	189	2773

Table 2: Di erences in quantities as a function of the time at which all interventions are enforced for the one-group model.

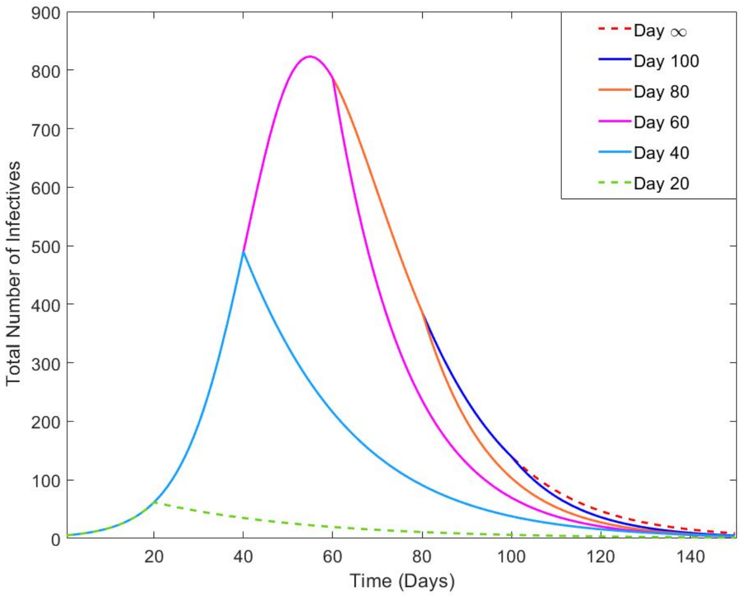

Figure 3: Variation in the total number of infected individuals as a function of the day that interventions are introduced into the one-group population.

7 Applications of the SIR Model for a Two-Group Heterogeneous Population

When there are two separate groups that respectively make two separate choices within a population, the SIR model can still be used to describe the transition rates between individuals in the infected, susceptible, and recovered categories. It is assumed in the six di erential equations that $b_1 = b_2$, where b_1 and b_2 represent the average infectious period of the disease for individuals in group one and group two respectively. Individuals in group one choose to disobey interventions. Individuals in group two choose to obey interventions. In the following equations, the term β_{ij} will be used to describe interaction between susceptibles in the ith group and infectives in the jth group, where i = 1, 2 and j = 1, 2.

$$\frac{dS_1}{dt} = \quad \beta_{11} \frac{S_1}{N_1} I_1 \quad \beta_{12} \frac{S_1}{N_1} I_2, \tag{5}$$

$$\frac{dI_1}{dt} = \beta_{11} \frac{S_1}{N_1} I_1 + \beta_{12} \frac{S_1}{N_1} I_2 \quad b_1 \; I_1, \tag{6}$$

$$\frac{dR_1}{dt} = b_1 \; I_1, \tag{7}$$

$$\frac{dS_2}{dt} = \quad \beta_{21} \frac{S_2}{N_2} I_1 \quad \beta_{22} \frac{S_2}{N_2} I_2, \tag{8}$$

$$\frac{dI_2}{dt} = \beta_{21} \frac{S_2}{N_2} I_1 + \beta_{22} \frac{S_2}{N_2} I_2 \quad b_2 \; I_2, \tag{9}$$

$$\frac{dR_2}{dt} = b_2 \; I_2. \tag{10}$$

The blue components represent the transition of individuals from S_1 to I_1. The orange components represent the transition of individuals from S_1 to I_2. The red components represent the transition of individuals from I_1 to $R_1 + R_2$. The green components represent the transition of individuals from S_2 to I_1. The cyan components represent the transition of individuals from S_2 to I_2. The magenta components represent the transition of individuals from I_2 to $R_1 + R_2$.

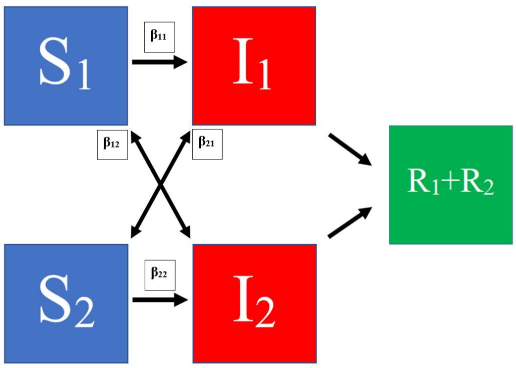

Figure 4: The transition of susceptible, infectious, and recovered individuals in the SIR two-group heterogeneous model.

The interactions between individuals in group one and group two are modelled using four beta values. These beta values are described in the following manner:

$$\beta_{11} = \beta \frac{N_1}{N} \tag{11}$$

$$\beta_{12} = \beta \frac{N_1}{N} \tag{12}$$

$$\beta_{21} = \beta \frac{N_2}{N} \tag{13}$$

$$\beta_{22} = \beta \frac{N_2}{N} \tag{14}$$

Equation 11 depicts the interaction between susceptible and infected individuals in group one. Equation 12 depicts the interaction between susceptible individuals in group one and infected individuals in group two. Equation 13 depicts the interaction between susceptible individuals in group two and infected individuals in group one. Equation 14 depicts the interaction between susceptible and infected individuals in group two.

Now, suppose that 60% of the population is initially in group one. This means that 60% of the contacts of an infectious individual in group one are with susceptibles in group one whilst 40% of their contacts are with susceptibles in group two. Similarly, the split of contacts for an infectious individual in group two is 60% with susceptibles in group one and 40% with susceptibles in group two. This scenario gives rise to the infectious contact rate matrix in equation 15.

The size of the population in group one and group two are respectively denoted as N_1 and N_2, with the total population simply being $N = N_1 + N_2$. There are four infection contact rates associated with the two-group heterogeneous model. We can represent these rates in the following matrix equation:

$$\underline{\beta} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \beta \begin{pmatrix} \frac{N_1}{N} & \frac{N_1}{N} \\ \frac{N_2}{N} & \frac{N_2}{N} \end{pmatrix} \tag{15}$$

8 Modelling Effect of Intervention Methods

In a two-group heterogeneous model, individuals in a population are split into two groups. Individuals from group one choose to disobey interventions. Individuals from group two choose to follow interventions. The di erent e ects of individual and combinations of interventions on the number of infectives in the population are explored with regards to the proportion of individuals in each group.

8.1 Level One Social Distancing

When both infectious and susceptible individuals in group one do not undergo social distancing, transmission of the disease is not reduced, resulting in $\beta_{11} = 1.0\ \beta$. When susceptible individuals in group one do not undergo social distancing but infected individuals in group two do undergo social distancing, transmission of the disease is reduced to 89.4% or 0.894, resulting in $\beta_{12} = 0.894 \quad \beta$. When susceptible individuals in group two undergo social distancing but infected individuals in group one do not undergo social distancing, transmission of the disease is reduced to 89.4% or 0.894, resulting in $\beta_{21} = 0.894 \quad \beta$. When both infectious and susceptible individuals in group two undergo social distancing, transmission of the disease is reduced to 80% or 0.80, resulting in $\beta_{22} = 0.8 \quad \beta$ [4, 7].

The infectious contact rate matrix for level one social distancing is expressed in the following manner:

$$\underline{\beta}_{\text{social}} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \beta \begin{pmatrix} \frac{N_1}{N} \quad 1.0 & \frac{N_1}{N} \quad 0.894 \\ \frac{N_2}{N} \quad 0.894 & \frac{N_2}{N} \quad 0.8 \end{pmatrix}$$

8.2 Proper Hand Hygiene

When both infectious and susceptible individuals in group one do not undergo proper hand hygiene, transmission of the disease is not reduced, resulting in $\beta_{11} = 1.0 \quad \beta$. When susceptible individuals in group one do not undergo proper hand hygiene but infected individuals in group two do undergo proper hand hygiene, transmission of the disease is not reduced as susceptible individuals will still be susceptible to the inward transmission of the disease, resulting in $\beta_{12} = 1.0 \quad \beta$. When susceptible individuals in group two undergo proper hand hygiene but infected individuals in group one do not undergo proper hand hygiene, transmission of the disease is reduced to 70% or 0.70, resulting in $\beta_{21} = 0.70 \quad \beta$. When both infectious and susceptible individuals in group two undergo proper hand hygiene, transmission of the disease is reduced to 70% or 0.70, resulting in $\beta_{22} = 0.70 \quad \beta$ [4, 9].

The infectious contact rate matrix for proper hand hygiene is expressed in the following manner:

$$\underline{\beta}_{\text{hygiene}} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \beta \begin{pmatrix} \frac{N_1}{N} \quad 1.0 & \frac{N_1}{N} \quad 1.0 \\ \frac{N_2}{N} \quad 0.7 & \frac{N_2}{N} \quad 0.7 \end{pmatrix}$$

8.3 Mask Enforcement

Wearing a cotton mask reduces outward transmission by 50% and reduces inward transmission by 20% [4]. When both infectious and susceptible individuals in group one do not wear masks, transmission of the disease is not reduced, resulting in $\beta_{11} = 1.0 \quad \beta$. $\beta_{12} = 0.5 \quad \beta$. When susceptible individuals do not wear masks but infected individuals do wear masks, transmission of the disease is reduced to 50% or 0.5, resulting in $\beta_{12} = 0.5 \quad \beta$. When susceptible individuals do wear masks but infected individuals do not wear masks, transmission of the disease is reduced to 80% or 0.8, resulting in $\beta_{21} = 0.8 \quad \beta$. When both infectious and susceptible individuals in group two do wear masks, transmission of the disease is reduced to 40% as $0.5 \quad 0.8 = 0.4$, resulting in $\beta_{22} = 0.4 \quad \beta$ [5, 6].

The infectious contact rate matrix for mask enforcement is expressed in the following manner:

$$\underline{\beta}_{\text{mask}} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \beta \begin{pmatrix} \frac{N_1}{N} & 1.0 & \frac{N_1}{N} & 0.5 \\ \frac{N_2}{N} & 0.8 & \frac{N_2}{N} & 0.4 \end{pmatrix}$$

8.4 All Interventions

When a combination of all interventions are introduced and both infectious and susceptible individuals do not follow interventions, transmission of the disease undergoes no reduction, resulting from $\beta_{11} = 1.0 \quad ((1.0 \quad (1.0 \; 1.0 \; 1.0))) \; \beta = 1.0 \; \beta$. When susceptibles do not follow interventions and infectives do, the transmission of the disease is reduced to 0.447, resulting from $\beta_{12} = 1 \quad ((1 \quad (0.894 \; 1.0 \; 0.5))) \; \beta = 0.447 \; \beta$. When susceptibles do follow interventions and infectives do not, the transmission of the disease is reduced to 0.5006, resulting from $\beta_{21} = 1 \quad ((1 \quad (0.894 \; 0.7 \; 0.8))) \; \beta = 0.5006 \; \beta$. When both infectives and susceptibles are following interventions, the transmission of the disease is reduced to 0.224, resulting from $\beta_{22} = 1 \quad ((1 \quad (0.8 \; 0.7 \; 0.4))) \; \beta = 0.224 \; \beta$. Here, the calculations assume that the e ectiveness of the interventions are independent of each other, where the intersection of the e ectiveness of the three interventions is being calculated.

The infectious contact rate matrix for all interventions is expressed in the following manner:

$$\underline{\beta}_{\text{all}} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} = \beta \begin{pmatrix} \frac{N_1}{N} & 1.0 & \frac{N_1}{N} & 0.447 \\ \frac{N_2}{N} & 0.5006 & \frac{N_2}{N} & 0.224 \end{pmatrix}$$

9 Results for the Two-Group Heterogeneous Population

Individuals in group one do not follow COVID-19 interventions and individuals from group two do follow interventions. Here, the di erent e ects of interventions on disease transmission are considered, depending on the number of individuals in each group.

9.1 Initial Conditions and Equations

For the two-group heterogeneous model, we rst de ne parameters of the model. The total population is $N = 3000$, the population in group one is N_1, the population in group two is N_2, the per-capita recovery rate is $b = 1/14$ days ^{1}, the basic reproduction number is $R_0 = 2.79$ [3]. There are a total of ve initial infected individuals chosen to be in group one, with $N_1 = 5$. However, if there are no individuals in group one due to the entire population being compliant with interventions, there are a total of ve initial infected individuals that are chosen to be in group two, with $N_2 = 5$. All interventions are xed to have been introduced at Day 20.

9.2 Summary of Results

For the two-group heterogeneous population, higher proportions of compliance among the population generally resulted in a decrease of all four quantities, which are the maximum number of infectives ($(I_1 + I_2)_{\text{max}}$), time to maximum infections (t_{max}), eradication time (t_{ext}), and total recovered individuals (t_{ext}). The reductions are dependent on the size of N_1, which represents the number of individuals in group one who disobey interventions. As the number of individuals in group one increased, the time to maximum infectives and the total number of recovered individuals increased. However, the more e ective the intervention, the more erratic the trend in the time to maximum infections and eradication time of the disease. This is demonstrated by a gradual increase followed by a gradual decrease of these two quantities when noncompliance with interventions increased. Figures and tables for the two-group heterogeneous model are placed in Appendix B.

10 Discussion

The SIR model is highly simplistic and models the spread of infectious diseases among a population with relative ease and accuracy. This is because the only parameters required to model simulations are the transmission rate β and the average infectious period 1/b. The main advantage here in using point estimates is that distributions for parameters are not required to be obtained in order for simulations to run, as distributions are hard to estimate and require enormous amounts of data sets in order to be generated.

One considerable limitation of the SIR model is that it assumes a homogeneous mixing of the population where all individuals in the population interact with each other evenly and make the same decisions. However, this limitation was overcome in the report as the two-group heterogeneous model was introduced, which took into account the ability for individuals to make di erent decisions to each other. The SIR model also assumes a closed population during the one year time simulation period of the COVID-19 pandemic, but this assumption would not have signi cantly lowered the accuracy of the simulation for this report as the number of births or deaths for a small town of 3000 individuals during the short course of a year can be deemed as negligible [2].

11 Conclusion

Masks are the most e ective individual intervention, followed by proper hand hygiene and level one social distancing. However, a combination of all three interventions is most e ective at reducing the transmission of contagious diseases. Additionally, earlier implementations of government interventions and higher compliance rates with these interventions are crucial for controlling the outbreak of a contagious disease, speci cally COVID-19. This highlights the importance behind understanding the idea of population heterogeneity and how di erences in compliance levels resulting from population heterogeneity can dramatically in uence and shape the outcome of a pandemic to be either forgiving or devastating. A potential future task is to calculate the threshold value for the number of disobeying individuals it would take for the basic reproduction number to be greater than one in the two-group heterogeneous model.

12 Appendix A: One-Group Homogeneous Population

The following gures and tables illustrate the relationship between the day that an intervention is introduced and the four investigated quantities in this report.

12.1 Effect of Level One Social Distancing $(m_i = 0.2)$

Level one social distancing is not an e ective intervention by itself, as it only reduces the basic reproduction of the disease by 20%, resulting in R_{social} = (1 0.2) R_0 = 2.232. Table 3 depicts that only (2773 2556)/2773 8% of the total population escapes infection when the population self-isolates at Day 20 as opposed to not self-isolating at all. Figure 5 shows that when level one social distancing is introduced, maximum infections and the total number of recovered individuals is only reduced by a relatively small extent as compared to the other interventions displayed in subsequent gures.

t_{social} (days)	I_{max}	t_{max}	t_{ext}	R_1
20	587	65	219	2556
40	667	55	204	2593
60	823	55	190	2688
80	823	55	185	2744
100	823	55	185	2746
1	823	55	189	2773

Table 3: Di erences in quantities as a function of the time at which social distancing is enforced for the one-group model.

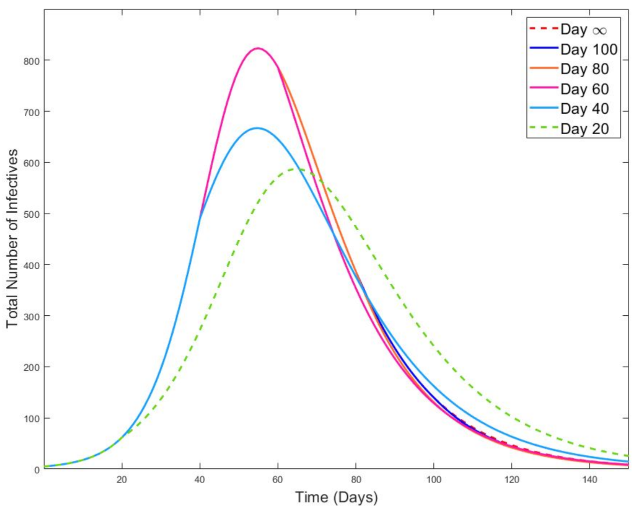

Figure 5: Variations in the total number of infected individuals as a function of the time at which social distancing enforced in the one-group population.

12.2 Effect of Proper Hand Hygiene ($m_i = 0.3$)

Proper hand hygiene reduces the basic reproduction number of the disease by 30%, resulting in R_{hygiene} = (1 0.3) R_0 = 1.953. Table 4 depicts that (2773 2363)/2773) 15% of individuals escape infection when the population follows the proper hand hygiene intervention at Day 20 as opposed to not following the proper hand hygiene intervention at all. Figure 6 shows that when proper hand hygiene is introduced before Day 40, maximum infections and the total number of recovered individuals is reduced to a greater extent than when level one social distancing was introduced. This is represented by the sharper decrease in gradients when the interventions are introduced at each particular day and a greater di erence between values in the rst and last row of values for maximum infections and total recovered individuals.

t_{hygiene}(days)	$I_{\max}$	$t_{\max}$	t_{ext}	R_1
20	452	72	244	2363
40	589	53	214	2449
60	823	55	189	2636
80	823	55	183	2729
100	823	55	183	2759
1	823	55	189	2773

Table 4: Di erences in quantities as a function of the time at which proper hand hygiene is enforced in the one-group population.

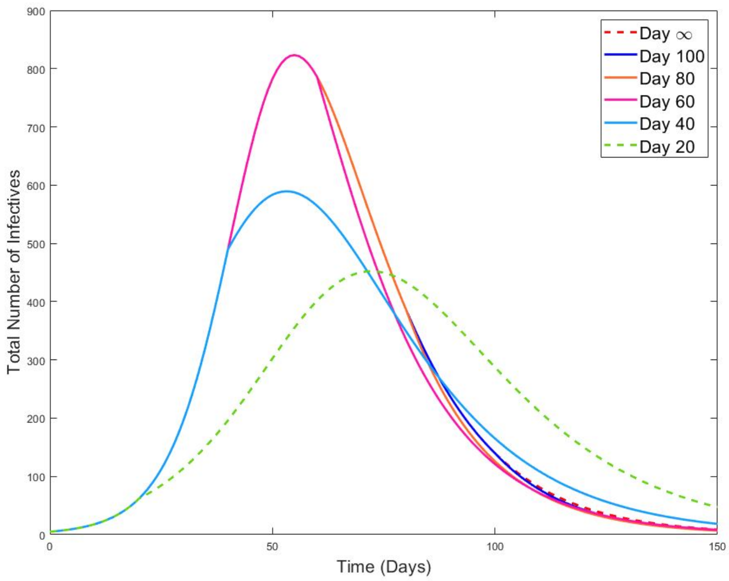

Figure 6: Variations in the total number of infected individuals as a function of the time at which proper hand hygiene is enforced in the one-group population.

12.3 Effect of Mask Enforcement $(m_i = 0.6)$

Mask enforcement is the most effective individual intervention. This can be noted by the most dramatic differences in maximum infections and total recovered individuals between the first and last row of Table 5, reducing the basic reproduction number by 60% and resulting in $R_{\text{mask}} = (1 - 0.6) R_0 = 1.116$. Table 5 also depicts that $(2773 - 868)/2773 \approx$ 69% of individuals escape infection when the population follows the mask enforcement intervention at Day 20 as opposed to not following the intervention at all. Figure 7 depicts the powerful effect that mask enforcement has in reducing the total number of infectives in the population, especially when introduced early on such as before Day 40. This is because the gradients stemming from the points at Day 20 and Day 40 are significantly reduced, meaning that a very low number of maximum infections is reached here. The minimal area under the curves for when masks are introduced before Day 40 also demonstrates the small quantity of total recovered individuals and validates masks as being highly effective in preventing disease transmission.

t_{mask} (days)	I_{max}	t_{max}	t_{ext}	R_∞
20	70	64	430	868
40	489	40	227	1687
60	823	55	180	2448
80	823	55	175	2684
100	823	55	177	2746
∞	823	55	189	2773

Table 5: Differences in quantities as a function of the time at which masks are enforced for the one-group model.

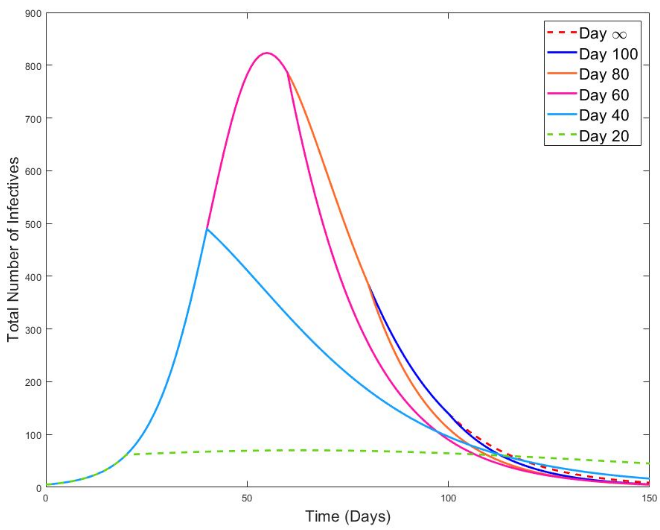

Figure 7: Variations in the total number of infected individuals as a function of the time at which masks are enforced in the one-group population.

13 Appendix B: Two-Group Heterogeneous Population

The following gures and tables illustrate changes in the four quantities when compliance levels change in the population.

13.1 Effect of Level One Social Distancing

Table 6 depicts that the greater the proportion of individuals who follow government interventions, the less the maximum infections and total recovered individuals. However, this comes at the expense of a longer time to maximum infections and a greater eradication time. This means that the disease dies out faster but infects more individuals when more people disobey interventions and dies out slower but infects less individuals when more people obey interventions.

Figure 8 depicts that the lower the proportion of individuals in group one, the lower the maximum infections, total recovered individuals, and the greater the time to maximum infections and eradication time. It is evident that social distancing is not a particularly e ective intervention by itself, as reductions in the curve are relatively minor compared to reductions formed by other interventions that are depicted in subsequent gures.

N_1	$(I_1 + I_2)_{max}$	t_{max}	t_{ext}	$(R_1 + R_2)_1$
0	587	65	219	2556
500	627	63	212	2560
1000	668	61	207	2641
1500	707	59	202	2679
2000	506	58	197	2713
2500	785	56	193	2744
3000	823	55	189	2773

Table 6: Variations in quantities as a function of the number of individuals in group one, when social distancing is enforced at Day 20.

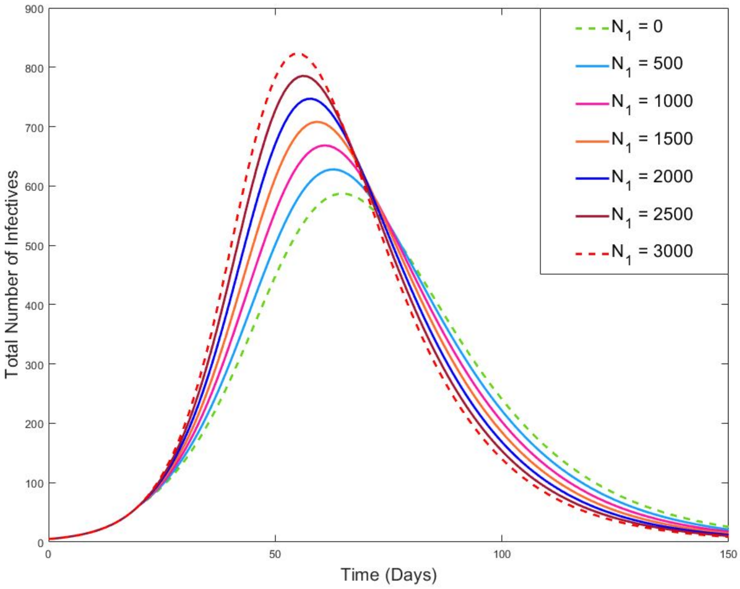

Figure 8: Variation in the total number of infected individuals as a function of the number of individuals in group one, when social distancing is enforced at Day 20.

13.2 Effect of Proper Hand Hygiene

Table 7 depicts a greater di erence between the rst and last row of values for the quantities of maximum number of infections and total recovered individuals respectively. These two quantities decrease in value when compliance levels increase. Figure 9 depicts similar trends to Figure 8, with magni ed reductions in the maximum number of infected individuals and the total number of recovered individuals.

N_1	$(I_1 + I_2)_{\text{max}}$	t_{max}	t_{ext}	$(R_1 + R_2)_1$
0	452	72	244	2363
500	513	68	231	2445
1000	574	65	220	2521
1500	636	62	210	2591
2000	698	59	202	2656
2500	760	57	195	2716
3000	823	55	189	2773

Table 7: Variations in quantities as a function of the number of individuals in group one, when proper hand hygiene is enforced at Day 20.

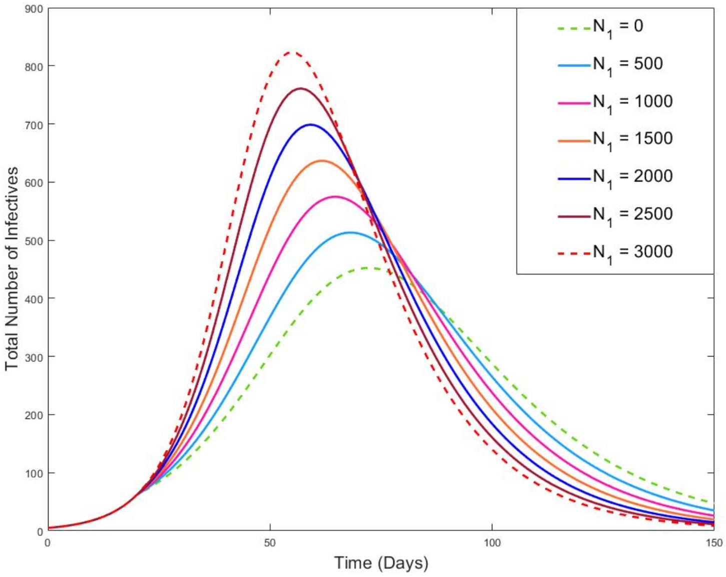

Figure 9: Variation in the total number of infected individuals as a function of the number of individuals in group one, when proper hand hygiene occurs at Day 20.

13.3 Effect of Mask Enforcement

Table 8 depicts a much more signi cant decrease in maximum infections and total recovered individuals than Table 6 and 7, when more individuals undergo compliance with interventions and are in group two instead of group one. The trend for the time to maximum infections has also become more erratic. This is observed when a decrease in compliance with interventions results in a sharp increase followed by a steady decrease in values for time to maximum infections. Figure 10 reinforces the observations drawn from Table 8, and also conveys that the peak of the curves and the total area under the curves are further reduced when compared to the weaker individual interventions of proper hand hygiene and level one social distancing.

N_1	$(I_1 + I_2)_{\text{max}}$	t_{max}	t_{ext}	$(R_1 + R_2)_1$
0	70	64	430	880
500	164	92	348	1553
1000	298	82	283	2009
1500	439	72	244	2312
2000	574	65	219	2520
2500	703	59	202	2667
3000	823	55	189	2773

Table 8: Variations in quantities as a function of the number of individuals in group one, when masks are enforced at Day 20.

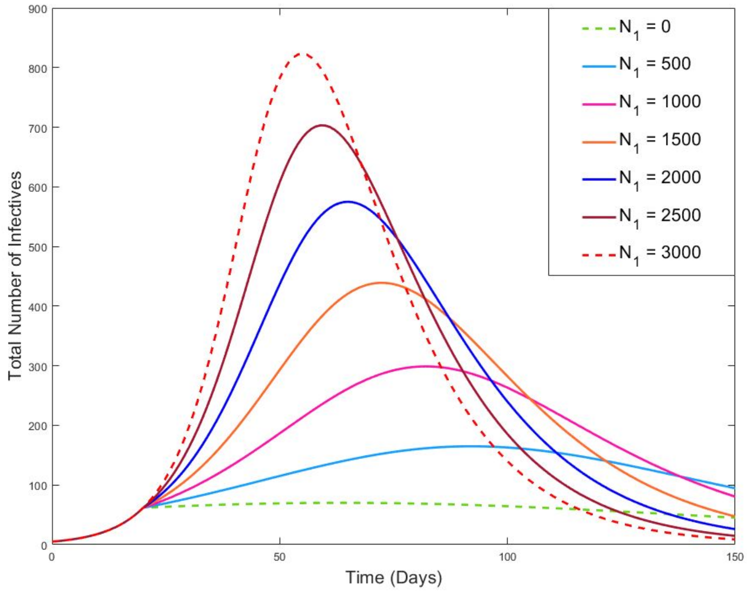

Figure 10: Variation in the total number of infected individuals as a function of the number of individuals in group one, when masks are enforced at Day 20.

13.4 Effect of All Three Interventions

The combination of all interventions creates the greatest reduction in maximum infections and total recovered individuals, as seen in Table 9 when comparing the rst and the last row of values. As the basic reproduction number is reduced below one when all interventions are introduced, this results in an erratic trend for the time to maximum infections and eradication time of the disease, as both quantities increase to a certain point before decreasing once again.

Figure 11 depicts that when everyone is in group two and follows interventions, a maximum number of infections is never formed after Day 20, the day that the intervention is introduced. This is because the basic reproduction number has fallen below one, resulting in an immediate negative gradient for the curve when $N_1 = 0$. There is also the smallest number of total recovered individuals possible for the simulations in this report when all interventions are introduced at Day 20, shown by the extremely small area under the green dotted curve. This conclusion demonstrates that when there is high level of compliance among the population and a combination of interventions are introduced at an early enough time, a minimal number of infections occur in the population, strongly bene ting the well-being of society and the healthcare system.

N_1	$(I_1 + I_2)_{\max}$	$t_{\max}$	t_{ext}	$(R_1 + R_2)_1$
0	61	20	162	186
500	61	20	361	479
1000	124	90	354	1258
1500	283	80	276	1866
2000	351	69	233	2277
2500	645	61	207	2565
3000	823	55	189	2773

Table 9: Variations in quantities as a function of the number of individuals in group one, when all interventions are enforced at Day 20.

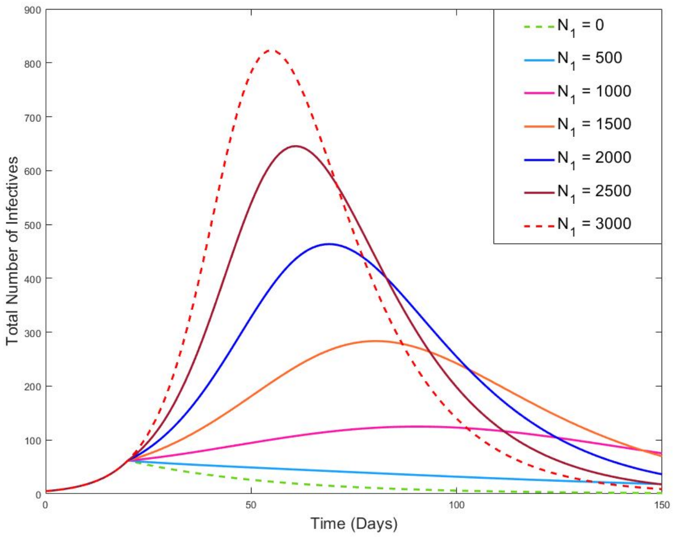

Figure 11: Variation in the total number of infected individuals as a function of the number of individuals in group one, when all interventions are enforced at Day 20.

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